GPS Positioning in a Multipath Environment

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***Abstract*—We address the problem of GPS signal delay estimation in a multipath environment with a low-complexity constraint. After recalling the usual early–late estimator and its bias in a multipath propagation context, we study the maximum-likelihood estimator (MLE) based on a signal model including the parametric contribution of reflected components. It results in an efficient algorithm using the existing architecture, which is also very simple and cheap to implement. Simulations show that the results of the proposed algorithm, in a multipath environment, are similar to these of the early–late in a single-path environment. The performance are further characterized, for both MLEs (based on the single-path and multipath propagation) in terms of bias and standard deviation. The expressions of the corresponding Cramér—Rao (CR) bounds are derived in both cases to show the good performance of the estimators when unbiased.**

***Index Terms*—Cramér–Rao bound, GPS, maximum-likelihood, multipath propagation, pseudo-random code, spread spectrum systems.**

I. Introduction

The global positioning system (GPS) allows everybody in the world, equipped with a GPS receiver, to determine their position (longitude, latitude, and altitude). The GPS application consists of estimation of the propagation duration of a known coded signal between an emitter satellite and the receiver. This delay measurement allows the computation of the distance between the user and the satellite, and with at least four estimates from the different emitters, the positioning can be realized; see, for instance, [3] and [4].

In this paper, we will consider only one signal from one emitter thanks to the weakness of the interferences between the different pseudo random codes (Gold codes) used by the GPS satellites (i.e., the satellites use almost orthogonal spread spectrum sequences [16]). For a given code, contributions from other satellites will be considered to be an additive Gaussian noise.

Under the additional condition of propagation by the sole line-of-sight path, the usual technics to solve the problem of delay estimation are issued from the maximum likelihood estimator (MLE), early–late receivers. They provide a good enough estimation to ensure the positioning in many applications and will show that their accuracy corresponds to the deterministic Cramér–Rao (CR) bound limit.

Unfortunately, multipath propagation can happen sometimes (urban environment, low-altitude flight) and generate important errors (up to 100 m) without the usual receiver to detect any problem. Enge [3] and others propose to narrow the correlators to reduce the bias in a tracking context. However, narrowing the correlator does not provide a good estimate when the propagation conditions change too quickly or when a first acquisition is required. As a result, many applications, using the GPS system, need other equipment (differential, GLONASS, etc. [4]) in order to reduce errors or prove validity of estimates. A method that determines the signal propagation delay in a multipath environment as well as in a single-path environment with the same accuracy as in the single-path case and without increasing the implementation cost will permit autonomy and reliability of the GPS system. To look for such a method, we consider the ML approach, taking into account the multipath propagation. Stoica and Nehorai [13] propose to improve the ML approach by considering a random model of the input sequence. However, in our context, this would involve a matrix inversion that cannot be allowed in terms of complexity. Improvements can also be obtained by refining the noise model in taking into account the receiving filter induced correlation. Again, this results in a matrix inversion to solve the likelihood maximization and, thus, cannot be considered in this study. The algorithm we propose is aimed to keep the complexity similar to that of the current early–late receiver.

After introducing some notations (Sections II and III), we recall in Section IV the ML estimator applied to a single-path model used by current receivers and its performance. An example illustrates why and how the presence of multipath propagation affects the estimation precision. Section V addresses the ML estimator, its performance, and the CR bound when applied to a two-path model (the direct signal $+$ one reflected signal). Results of this estimator (called ML2) are compared with the current ML estimator, and improvements of precision are shown. Finally, we propose to extend the two-path estimator (ML2) to the general case of multipath propagation.

II. Spread Spectrum Code Properties

The emitted signal is generated by sending periodically a well-chosen pseudo-random sequence $G[n]$ through a lowpass shaping filter $h(t)$. $G[n]$ is generated as a Gold code (see [7]) whose properties are recalled in the following. The sequence is

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composed by M binary chips of time duration T; the sequence period is then $T_m = MT$. Before pulse shaping, the signal denoted as $G(t)$ can be expressed as

$$G(t) = \sum_n G[n] U_T(t - nT) \quad (1)$$

where $U_T(t)$ takes the value of 1 for t in $[0, T]$ and 0 elsewhere. Thus, the emitted signal $c(t)$ after the pulse-shaping filter h (of bandwidth B around 4 MHz) of $G(t)$ is then

$$c(t) = \int G(u) h(t - u) du. \quad (2)$$

This filter is assumed to provide an ideal pulse shaping. We explain, in the sequel, how time delay estimation is actually realized, yet we can introduce that the maximum likelihood estimator uses properties of the crosscorrelation between the Gold code and received signal. As a result, the autocorrelation properties of Gold codes are very important to understand GPS methods. If the autocorrelation of $c(t)$ is denoted by $\Phi(\tau) = \int_{T_m} c(t)c(t-\tau)dt$, (4) describes the properties of this function.

$$\Phi(\tau) = \int_{T_m} c(t)c(t - \tau)dt \quad (3)$$

$$\Phi(\tau) = \begin{cases} 1 - \frac{M+1}{M}\left|\frac{\tau}{T}\right| \simeq 1 - \left|\frac{\tau}{T}\right|, & \text{for } |\tau| \leq T \\ -\frac{1}{M} \simeq 0, & \text{for } |\tau| > T. \end{cases} \quad (4)$$

The approximations above are justified for the actual high values of M ($M = 1023$, $T = 977.5$ ns and $T_m = 1$ ms in civil GPS applications). The main advantage to using the crosscorrelation function is in improving the signal-to-noise ratio (SNR). Indeed, for an integration time (the crosscorrelation is computed during this time) of 1 ms (equal to the code period T_m), the SNR is improved by $+30$ dB (the signal is sampled at the rate $F_e = 2$ MHz) and more if the integration time is greater. Such gains let us deal with SNR (before integration) of around -20 to -30 dB.

III. Multipath Propagation Model

In mobile satellite communication, multipath propagation is often modeled by a Rician distribution; see [6]. The received signal is then supposed to be composed by the LOS signal component and by the sum of P reflected signals whose amplitudes a_k, delays θ_k, frequency Doppler f_k, and phases ϕ_k differ from the LOS signal parameters (A, τ, f_d, $\phi = 0$). If the frequency demodulator is assumed to be locked on the LOS path frequency f_d, (5) can be used to modelize the baseband received signal

$$s(t) = Ac(t-\tau) + \sum_{k=1}^{P} a_k c(t - \theta_k) e^{-2\pi j \Delta_{f_k} t + j\phi_k} + b(t). \quad (5)$$

Moreover, differences between Doppler frequencies f_k and f_d are generally very small since the distance and relative speed between the mobile and reflecting surface are very small with respect to the satellite-mobile ones; see [1]. As a result, where $\Delta_{f_k} = (f_k - f_d)$ is considered to be constant and included in the reflected components, phase shifts ϕ_k in (6) also depend on the reflecting surface nature and on the angle of incidence of the kth signal.

$$s(t) = A(t)c(t - \tau) + \sum_{k=1}^{P} a_k(t) c(t - \theta_k) e^{j\phi_k} + b(t) \quad (6)$$

where
- $c(t)$ emitted signal (assumed known);
- $A(t)$ line-of-sight signal amplitude;
- τ received signal lag to be estimated.

Characterizations of phase shifts ϕ_k effects on direct signal delay ML estimation (see [2] and [11]) allow us to conclude that the worst configuration for estimation accuracy is when the direct and reflected signals are in phase ($\phi_k = 0°$ or 180°). On the contrary, a reflected signal in quadrature ($\phi_k = \pi/2$) does not generate a biased estimation. As a result, from now on, we will only consider that signals are always in phase. Therefore, we neglect information that could be in the quadrature component by projection. Moreover, we can guarantee minimal performance since all reflections being in-phase corresponds to the worst possible case. Thanks to the different pseudo random Gold codes used by the satellites, the remaining interfering signal after crosscorrelation is modelized by the Gaussian and real-valued noise $b(t)$ that includes the thermal noise.

Sampled data are collected during N_p symbols ($T_o = N_p T_m$), where the time delay τ is assumed to be constant, whereas the amplitude $A(t)$ is a constant function (7) on time periods of only T_m. In fact, this amplitude could be set to a constant value on intervals of $N_p T_m$, but for robustness reasons, this hypothesis has been preferred. The sampling frequency is typically $F_e = 1/T_e = 2$ MHz and $N_p = 100$ for a SNR around -20 dB. Finally, N is the number of code samples during T_m $(T_m = MT = NT_e)$.

$$A(t) = \sum_{i=1}^{N_p} A_i U_{T_m}(t - (i-1)T_m), \text{ for } t \in [0, T_o] \quad (7)$$

where $U_{T_m}(t)$ takes the value 1 for t in $[0; T_m]$ and 0 elsewhere.

IV. Early–Late Receiver

In this section, we recall the usual GPS receiver that was conceived assuming the propagation is due to the line-of-sight path only: the so-called early–late receiver. We also recall its lack of robustness to a strong multipath scatterer inducing a delayed path within the length of the spreading sequence.

A. Early–Late Receiver

In the single path propagation, the frequency demodulation is supposed to be perfect; thus, (6) simplifies to

$$s(t) = A(t)c(t - \tau) + b(t). \quad (8)$$

Denoting $\underline{A} = [A_1, \ldots, A_{N_p}]$, the probability density function is

$$p(s, [\tau, \underline{A}]) \propto \exp\left\{-\frac{1}{2\sigma^2} \sum_{i=1}^{N_p} \int_{(i-1)T_m}^{iT_m} (s(t) - A_i c(t-\tau))^2 \, dt\right\}. \quad (9)$$

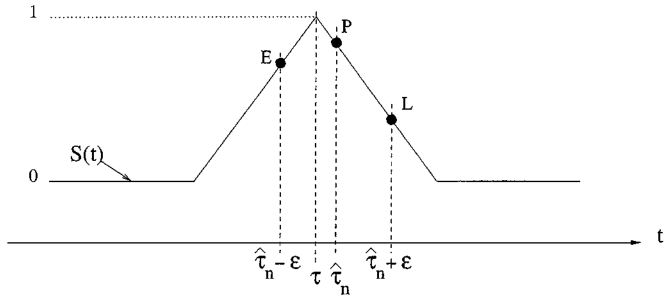

Fig. 1. Configuration of points E, P, and L.

The MLE zeros the derivative of the log-likelihood function (9)

$$\begin{cases} \frac{\partial p(s,[\tau,\underline{A}])}{\partial \tau} = 2\sum_{i=1}^{N_p} \hat{A}_i \frac{\partial S_i(\hat{\tau})}{\partial \tau} = 0 \\ \frac{\partial p(s,[\tau,\underline{A}])}{\partial \hat{A}_i} = 2S_i(\hat{\tau}) - 2\rho_c \hat{A}_i = 0, \quad \forall\, i \in [0; N_p] \end{cases} \quad (10)$$

where ρ_c and $S_i(\tau)$ represent, respectively, the energy of the filtered pseudo random code $c(t)$ on one period T_m and the cross-correlation between the input signal $s(t)$ and code $c(t)$ during the interval $[(i-1)T_m, iT_m]$:

$$\rho_c = \int_{T_m} c^2(t)dt, \quad S_i(\tau) = \int_{(i-1)T_m}^{iT_m} s(t)c(t-\tau)dt. \quad (11)$$

Replacing $\hat{A}_i$ by $\hat{A}_i(\hat{\tau}) = S_i(\tau)/\rho_c|_{\tau=\hat{\tau}_0}$, (10) reduces in solving $\sum_{i=1}^{N_p} S_i(\tau)\partial S_i(\hat{\tau})/\partial\tau = 0$. Since τ is a nonlinear parameter of the crosscorrelation functions S_i, it is impossible to give an explicit expression of its optimal value. To solve this problem, an iterative MLE can be computed (see [5] for the iterative Newton algorithm):

$$\hat{\tau} = \hat{\tau}_0 - \frac{\sum_{i=1}^{N_p} S_i(\hat{\tau}_0)\frac{\partial S_i(\hat{\tau}_0)}{\partial \tau}}{\sum_{i=1}^{N_p} S_i(\hat{\tau}_0)\frac{\partial^2 S_i(\hat{\tau}_0)}{\partial \tau^2} + \sum_{i=1}^{N_p}\left(\frac{\partial S_i(\hat{\tau}_0)}{\partial \tau}\right)^2}. \quad (12)$$

The second term of the denominator of (12) can be neglected; see [11]. In order to implement (12), we need an estimation of S_i and its derivatives. Since $s(t)$ is sampled at $F_e = 1/T_e = 2$ MHz and the number of samples collected during a time T_m is N $(T_m = NT_e)$, one access the values $S_i[\tau + kT_e] = \sum_{l=1}^{N} s_i[lT_e]c[\tau+(k-l)T_e]$ spaced by a time interval $T_e \simeq T/2$ only. The algorithm is initialized with a precision of about $T/2$ (through maximization of $S_i[kT_e]$), which is assumed to be correct in the sequel. The updating equation is also implemented as the so-called early–late algorithm:

$$\hat{\tau}_{n+1} = \hat{\tau}_n - \frac{\sum_{i=1}^{N_p}(E_i(n) - L_i(n))P_i(n)}{4\sum_{i=1}^{N_p} P_i^2(n)} T \quad (13)$$

where $P_i(n)$, $E_i(n)$, and $L_i(n)$, respectively, denote $S_i(\hat{\tau}_n)$, $S_i(\hat{\tau}_n - \varepsilon)$, and $S_i(\hat{\tau}_n + \varepsilon)$. Equation (13) is obtained using the approximation $E_i(n)+L_i(n)-2P_i(n) \simeq (-2\varepsilon P_i(n)/T)$. Fig.1 presents the configuration of the three points E, P, and L on the noiseless crosscorrelation function described in Section II.

B. Performance of the Early–Late Receiver

The performance of the early–late receiver have already been investigated through simulations in many papers; see, for instance, [14]. For the sake of comparaison, we recall the performance of the early–late receiver when the single-path model (8) is valid and derive the corresponding CR bound for its standard deviation. We explain also shorthy the bias caused when multipath propagation occurs.

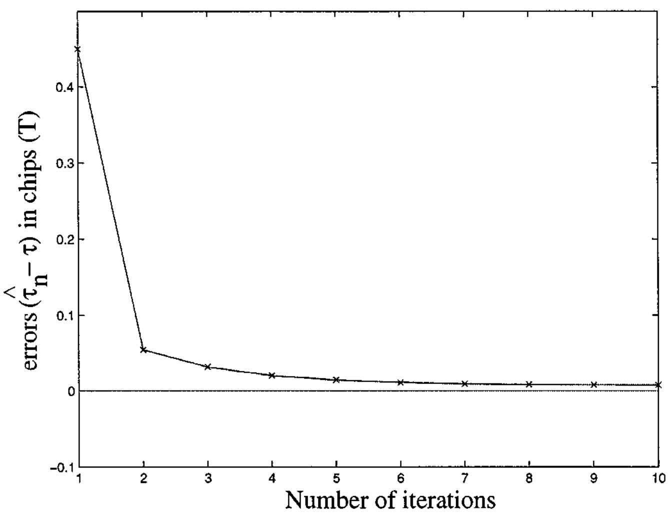

Fig. 2. Single-path propagation, SNR $= -20$ dB, $N_p = 100$. Errors in chips (normalized) versus iterations.

Performance for a Single-Path Propagation: Assuming there is no estimation bias when the single-path model (8) is valid, the estimator variance is minimized by the CR bound [5]. The corresponding Fisher information matrix of all parameters is derived in Appendix, and its expression is displayed in (14). The first line concerns delay τ parameter, and the N_p others lines are relative to amplitude A_i parameters. Equation (14) is obtained under the following hypothesis $B \gg 1/T_m$, where B is the lowpass receiver filter bandwidth. This condition is valid since for the GPS application, $1/T_m = 1$ kHz, and $B \approx$ MHz.

$$F = \left[\begin{array}{c|ccc} w & 0 & \cdots & 0 \\ \hline 0 & & & \\ \vdots & & \lambda I_{N_p \times N_p} & \\ 0 & & & \end{array}\right] \quad (14)$$

where $I_{N_p \times N_p}$ is the identity matrix of size $(N_p \times N_p)$, and

$$w = \frac{4\pi^2}{\sigma^2}\sum_{i=0}^{N_p-1} A_i^2 \int_{-B/2}^{B/2} f^2\bar{\Phi}(f)df$$
$$\lambda = \frac{1}{\sigma^2}\int_{-B/2}^{B/2} \bar{\Phi}(f)df \quad (15)$$

where $\bar{\Phi}$ is the Fourier transform of the code correlation function in (4). Then, the CR bound associated twith the delay estimation is w^{-1}.

Simulations show that the series $\hat{\tau}_n$ (see Fig. 2) converges to τ within a small number of iterations. Computing the bias of the ML estimator averaged on 50 realizations for SNR from -20 dB to -30 dB, the bias was around 0.001 chips. This result confirms that the ML estimator is unbiased in the case of a single-path propagation. The estimation variance computed in the same conditions is very close to the CR bound (see Fig. 3)

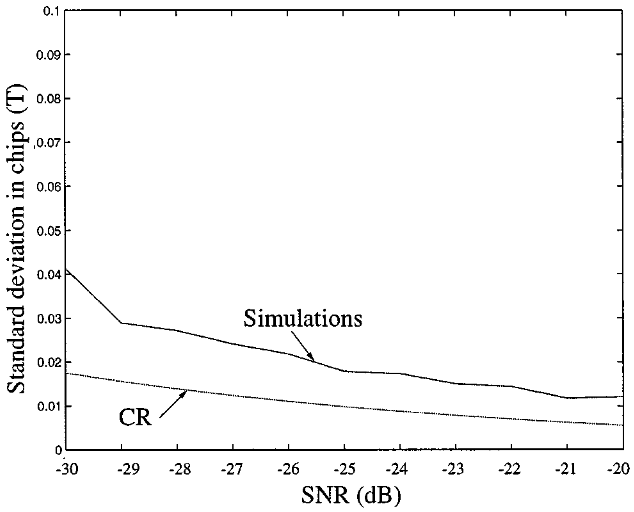

Fig. 3. Single-path propagation: Standard deviation and CR bound versus SNR.

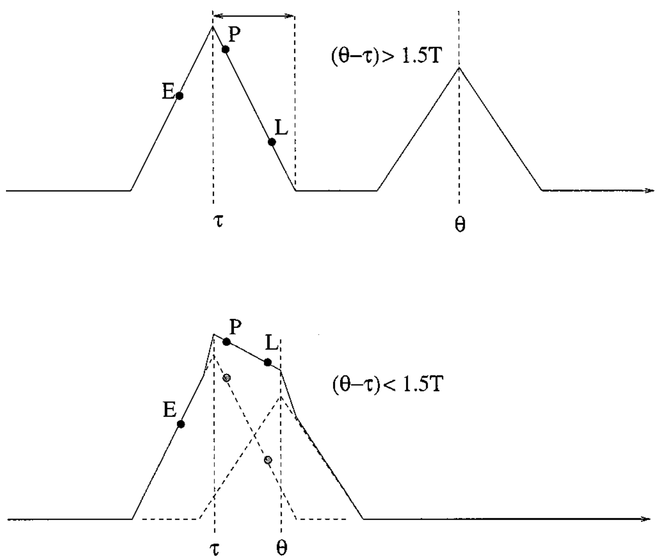

Fig. 4. Crosscorrelation of $s(t)$ in a two-path context for two configurations of $\theta - \tau$.

(less than 0.02 difference). The parameters used for the simulation ($N_p = 100$ and $\mathrm{SNR} = -20$ dB) were set to match a range of values encountered in typical GPS applications.

Performance for a Multipath Propagation: It is well known that the early–late receiver is biased when there is multipath propagation [14], specifically when the scaterers induce a supplementary delay θ with $\tau < \theta < 1.5T$. The reason for this bias is illustrated in a two-path propagation context (16) in Fig. 4. This error is easily explained by the values of P, where L is modified by the second signal contribution to the crosscorrelation function. This problem is due to a mismatch between the single-path signal model and the real multipath data. Simulations show that the early–late estimator bias can be up to $0.3T$, for a *direct to reflected signals ratio* (DRR) of 5 dB. $0.3T$ corresponds to an estimation error of 100 m in distance. Therefore, in the sequel, we study the direct signal delay ML estimator based on a multipath model.

V. Two-Path GPS Receiver

We want to propose a method to estimate the direct signal propagation delay in the presence of P reflected paths with P greater than or equal to 1. First, we consider the case of 1 reflected path for which we derive the ML receiver and study its performance including the corresponding CR bound. A very strong constraint in this study is the implementation cost. We want to keep it similar to that of the early–late receiver.

A. Two-Path Model

The two-path model is composed by a direct and a reflected signal both in phase with respective delays (τ, θ) and amplitudes (A, a). $b(t)$ is an additive Gaussian noise:

$$s(t) = A(t)c(t-\tau) + a(t)c(t-\theta) + b(t). \qquad (16)$$

We assume that $A(t)$ is a constant function on time intervals of T_m and that the same applies for $a(t)$. Our goal is to estimate the direct signal delay τ by an MLE based on model (16). This estimator is denoted ML2 estimator in reference to the two-path model. To be efficient, the proposed algorithm needs to estimate correctly the direct signal delay when there is one reflected signal (as in the two-path model) but when there is also no reflected component (single-path model for $s(t)$).

B. New Likelihood Function $V([\tau, \theta, \underline{A}, \underline{a}])$

Using model (16), the expression of the density of probability to maximize is $p(s/[\tau, \underline{A}])$, which is proportional to

$$\exp\left\{-\frac{1}{2\sigma^2}\int_{T_o}\left(s(t) - A(t)c(t-\tau) - a(t)c(t-\theta)\right)^2 dt\right\}. \qquad (17)$$

As in Section IV, the previous section, and the hypothesis on $a(t)$, the log-likelihood function $V([\tau, \theta, \underline{A}, \underline{a}])$ is expressed in (18). We note an expression similar to the single-path one (terms related to τ and A_i). Similar terms are found for the reflected signals (θ, a_i). Finally, a crosscorrelation term between direct and reflected signals appears.

$$V(\tau, \theta, \underline{A}, \underline{a}) = -\int_0^{T_o} s^2(t)dt + 2\sum_{i=1}^{N_p} A_i S_i(\tau) - \rho_c \sum_{i=1}^{N_p} A_i^2 + 2\sum_{i=1}^{N_p} a_i S_i(\theta) - \rho_c \sum_{i=1}^{N_p} a_i^2 - 2\Phi(\theta - \tau)\sum_{i=1}^{N_p} A_i a_i. \qquad (18)$$

C. Maximum Likelihood ML2 Estimator

The ML2 estimator $(\hat{\tau}, \hat{\theta}, \underline{\hat{A}}, \underline{\hat{a}})$ zeroes the derivative of the likelihood function (18), which is denoted $V(.)$ as (19), shown at the bottom of the next page.

As previously stated, estimates of the delays τ and θ are nonexplicit values because they appear as parameters of crosscorrelation and autocorrelation functions $S_i(.)$ and $\Phi(.)$. To get an expression of the direct signal delay $\hat{\tau}$ in this context, application of Taylor's formula and Newton algorithm could be done with $\underline{X}$ now being a vector of delays and amplitudes $[\hat{\tau}, \hat{\theta}, \underline{\hat{A}}, \underline{\hat{a}}]$.

Unfortunately, even if the second derivative terms of the likelihood function $\partial^2 V(.)/\partial\tau^2, \partial^2 V(.)/\partial\theta^2, \ldots$ can be easily derived, it turns to be impossible to get an explicit expression of the inverse Hessian matrix $\partial^2 V(\underline{X})/\partial\underline{X}^2$.

To solve this problem, we propose to rewrite the first two equations of (19) as

$$\begin{cases} \frac{\partial V(.)}{\partial\tau} = 2\sum_{i=1}^{N_p} \hat{A}_i \frac{\partial}{\partial\tau}[S_i(\hat{\tau}) - \hat{a}_i\Phi(\hat{\theta}-\hat{\tau})] \\ \frac{\partial V(.)}{\partial\theta} = 2\sum_{i=1}^{N_p} \hat{a}_i \frac{\partial}{\partial\theta}[S_i(\hat{\theta}) - \hat{A}_i\Phi(\hat{\theta}-\hat{\tau})]. \end{cases} \tag{20}$$

Using the expressions of the autocorrelation $\Phi(.)$, (3) and the crosscorrelation $S_i(.)$ (11), (20) can be rewritten as

$$\frac{1}{2}\frac{\partial V(.)}{\partial\tau} = \sum_{i=1}^{N_p} \hat{A}_i \frac{\partial S_i'(\hat{\tau})}{\partial\tau} = 0 \tag{21}$$

$$\frac{1}{2}\frac{\partial V(.)}{\partial\theta} = \sum_{i=1}^{N_p} \hat{a}_i \frac{\partial S_i'(\hat{\theta})}{\partial\theta} = 0 \tag{22}$$

where

$$\begin{cases} S_i'(\hat{\tau}) = \int_{(i-1)T_m}^{iT_m} \left[s(t) - \hat{a}_i c(t-\hat{\theta})\right] c(t-\hat{\tau})\,dt \\ S_i''(\hat{\theta}) = \int_{(i-1)T_m}^{iT_m} \left[s(t) - \hat{A}_i c(t-\hat{\tau})\right] c(t-\hat{\theta})dt. \end{cases} \tag{23}$$

The function $S_i'(\hat{\tau})$ is the crosscorrelation between the replica code delayed by $\hat{\tau}$ and the received signal minus the reflected signal $a_i c(t-\hat{\theta})$. If a_i and $\hat{\theta}$, which are the reflected signal parameters, are known, the crosscorrelation $S_i'(\hat{\tau})$ can be computed, and solving (21) is equivalent to solve the problem of delay τ estimation in a single-path propagation. Indeed, (21) would be equal to (10) found in paragraph Section IV-A. Accordingly, (22) is equivalent to the delay θ estimation if A_i and $\hat{\tau}$, which are the direct signal parameters, are assumed to be known.

Therefore, we propose a new algorithm (denoted as ML2 in the sequel) consisting of realizing, iteratively, both estimations using for each the previous ML estimator calculated with the parameters from the previous iteration. Note that this algorithm is first introduced in [15] as an *ad hoc* algorithm to combat multipath with details of neither the implementation nor the performance study. Each track needs the amplitude estimation $\underline{\hat{A}}_n$ and $\underline{\hat{a}}_n$ to compute signals $[s(t) - \hat{a}_{i,n}c(t-\hat{\theta}_n)]$ and $[s(t) - \hat{A}_{i,n}c(t-\hat{\tau}_n)]$ used for (23). The expressions of these estimators can be deduced from the third and fourth lines of (19)

$$\begin{cases} \hat{A}_{i,n+1} = \frac{\rho_c S_i(\hat{\tau}_n) - \Phi(\hat{\theta}_n - \hat{\tau}_n) S_i(\hat{\theta}_n)}{\rho_c^2 - \Phi(\hat{\theta}_n - \hat{\tau}_n)^2} \\ \hat{a}_{i,n+1} = \frac{\rho_c S_i(\hat{\theta}_n) - \Phi(\hat{\theta}_n - \hat{\tau}_n) S_i(\hat{\tau}_n)}{\rho_c^2 - \Phi(\hat{\theta}_n - \hat{\tau}_n)^2}. \end{cases} \tag{24}$$

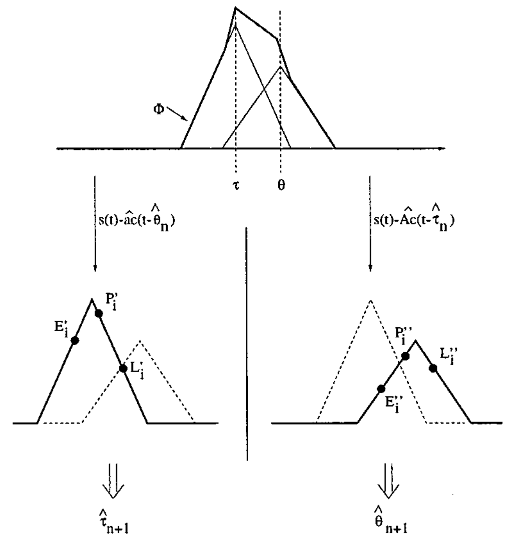

Fig. 5. ML2 estimator principle.

D. ML2 Estimator Implementation

We denote $\hat{\tau}_n$ and $\hat{\theta}_n$ as the previous iteration estimates of the delay parameters. During integration time, the following crosscorrelations are computed: $S_i(t-\hat{\tau}_n)$, $S_i(t-\hat{\theta}_n)$ as in (11) and $S_i'(t-\hat{\tau}_n)$, $S_i'(t-\hat{\tau}_n-\varepsilon)$, $S_i'(t-\hat{\tau}_n+\varepsilon)$, $S_i''(t-\hat{\theta}_n)$, $S_i''(t-\hat{\theta}_n-\varepsilon)$ and $S_i''(t-\hat{\theta}_n+\varepsilon)$ as in (23) with, respectively, in each case, the last estimates of the amplitudes given by (24).

Compared to ML, five additional integrators are used by the ML2 algorithm. However, these integrations are realized simultaneously so that ML2 and ML processing delays are equal. On one hand, early, late, and prompt values of crosscorrelations $P_i'(n) = S_i'(t-\hat{\tau}_n)$, $E_i'(n) = S_i'(t-\hat{\tau}_n-\varepsilon)$ and $L_i'(n) = S_i'(t-\hat{\tau}_n+\varepsilon)$ are used to compute $\hat{\tau}_{n+1}$:

$$\hat{\tau}_{n+1} = \hat{\tau}_n - \frac{\sum_{i=1}^{N_p}\left(E_i'(n) - L_i'(n)\right)P_i'(n)}{4\sum_{i=1}^{N_p} P_i'^2(n)}T. \tag{25}$$

On the other hand, early, late, and prompt values of crosscorrelations $P_i''(n) = S_i'(t-\hat{\theta}_n)$, $E_i''(n) = S_i'(t-\hat{\theta}_n-\varepsilon)$, and $L_i''(n) = S_i'(t-\hat{\theta}_n+\varepsilon)$ are used to compute $\hat{\theta}_{n+1}$:

$$\hat{\theta}_{n+1} = \hat{\theta}_n - \frac{\sum_{i=1}^{N_p}\left(E_i''(n) - L_i''(n)\right)P_i''(n)}{4\sum_{i=1}^{N_p} P_i''^2(n)}T. \tag{26}$$

Fig. 5 shows positions of points E_i', L_i', P_i', E_i'', L_i'', and P_i'' at the instant n in comparison with the crosscorrelation function $\Phi(\tau)$ between the noiseless incoming signal and replica code. Both delays are estimated separately using parameters $(\hat{A}, \hat{a})$, which are equal to the last estimates $(\hat{A}_{i,n}, \hat{a}_{i,n})$ that have been

$$\begin{cases} \frac{\partial V(.)}{\partial\tau} = 2\sum_{i=1}^{N_p} \hat{A}_i \frac{\partial S_i(\hat{\tau})}{\partial\tau} - 2\frac{\partial\Phi(\hat{\theta}-\hat{\tau})}{\partial\tau}\sum_{i=1}^{N_p} A_i a_i = 0 \\ \frac{\partial V(.)}{\partial\theta} = 2\sum_{i=1}^{N_p} \hat{a}_i \frac{\partial S_i(\hat{\theta})}{\partial\theta} - 2\frac{\partial\Phi(\hat{\theta}-\hat{\tau})}{\partial\theta}\sum_{i=1}^{N_p} A_i a_i = 0 \\ \frac{\partial V(.)}{\partial\hat{A}_i} = 2S_i(\hat{\tau}) - 2\rho_c\hat{A}_i - 2a_i\Phi(\hat{\theta}-\hat{\tau}) = 0, \qquad \forall\, i \in [0; N_p] \\ \frac{\partial V(.)}{\partial\hat{a}_i} = 2S_i(\hat{\theta}) - 2\rho_c\hat{a}_i - 2A_i\Phi(\hat{\theta}-\hat{\tau}) = 0, \qquad \forall\, i \in [0; N_p]. \end{cases} \tag{19}$$

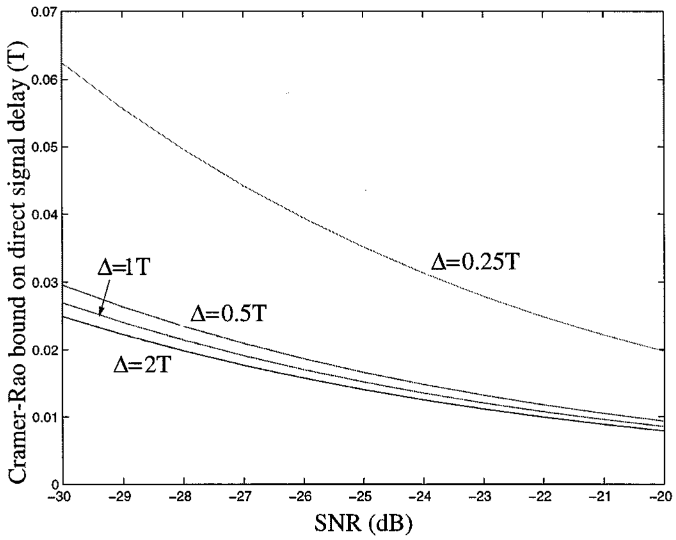

Fig. 6. CR bound of direct signal delay estimator versus SNR for various $\Delta = (\theta - \tau)$.

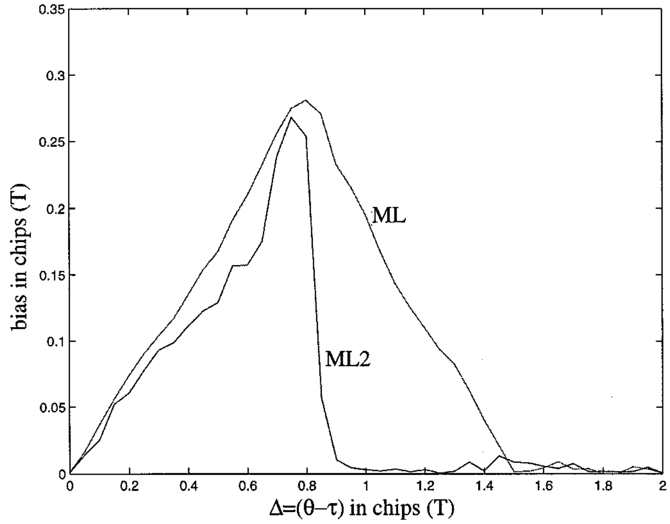

Fig. 7. Bias versus Δ, $\varepsilon = 1.0T$.

computed just previously. The global estimator architecture has been patented [12].

E. CR Bound

In the case of a two-path propagation model, the Fischer's information matrix is no longer diagonal. As for the likelihood function, crosscorrelation terms between the direct and reflected signals appear, and it turns to be more difficult to get values of CR bounds; they have, however, been derived (see the Appendix for the expression).

Indeed, the CR bound values depend on noise and direct and reflected signal powers. Moreover, according to Appendix A–2 (50), they depend on the paths delays difference $\Delta = (\theta - \tau)$. Fig. 6 shows the CR bound of direct signal delay estimation versus SNR for several values of Δ for a DRR of 5 dB. When the direct and reflected signals get close to each other, the CR bound increases because it becomes too difficult to separate the two contributions, whereas when Δ is greater than $1.5T$, the CR bound of the two-path model becomes equal to the CR bound found in the case of a single-path model. Indeed, for high values of Δ, cross correlations between the reflected signal and direct signal are almost zero thanks to properties of pseudo-random Gold codes.

F. ML2 Algorithm Constraints and Initialization

The initialization of the first ML2 estimator $\hat{\tau}$ is the same as in the case of the ML estimator. However, the initialization of the second estimator $\hat{\theta}$ is much more difficult. Indeed, when the ML2 algorithm is initialized, no *a priori* information on the reflected signal is available, and even its presence is uncertain. Since results show that the need for estimating is effective when θ in $[\tau,\ \tau + 1.5T]$, $\hat{\theta}_n$ is forced to be

$$\hat{\tau}_n < \hat{\theta}_n \leq \hat{\tau}_n + 1.5T \tag{27}$$

and arbitrarily, $\hat{\theta}_0$ is equal to $(\hat{\tau}_0 + 0.75T)$. A reinitialization is performed each time $\hat{\theta}_n$ does not respect (27). Finally, a detection of the presence of the reflected signal is done by a test on the amplitudes $\hat{A}$ and $\hat{a}$. In fact, when the direct to reflected signal power ratio 20 $\log(A_i/a_i)$ is greater than 12 dB, the influence of the reflected signal is close to zero; therefore, the amplitude estimate $\hat{a}_i$ is forced to zero. As a result, when a_i/A_i is close to zero, ML2 algorithm is reduced to the ML algorithm.

G. ML2 Estimator Performances in Two-Path Propagation

The incoming signal $s(t)$ is generated by the two-path propagation model (16). Fig. 7 compares the ML and ML2 bias versus $\Delta = (\theta - \tau)$ in $[0; 2T]$ and with a correlation window of $\varepsilon = 1T$. For Δ in $[1.5T; 2T]$, we verify that the ML and ML2 algorithms are equivalent. For Δ in $[0.9T; 1.5T]$, the proposed algorithm is successful in estimating the direct signal delay (almost no bias), whereas the ML algorithm is biased. Finally, for Δ in $[0T; 0.9T]$, the two algorithms are quite equivalent again, but they are both biased. In each case, the test on amplitude has been able to detect correctly the presence or absence of a reflected signal in $[\tau; \tau + 1.5T]$. Results have been greatly improved, but some bias still remains for Δ in $[0; 0.9T]$.

Additional improvement can be obtained by narrowing the correlator (smaller values of ε; see [3]). This technique is often used to reduce estimation errors in presence of reflected signals but is insufficient by itself. Fig. 8 displays the same simulation, only ε was decreased to $0.5T$. The bias of the ML algorithm is indeed reduced but remains important, whereas the ML2 algorithm results are satisfactory since they remove almost entirely the estimation bias for Δ greater than $0.3T$. Some errors remain for Δ in $[0; 0.3T]$, but they are not too large.

Finally, Fig. 9 compares the CR bound and standard deviation of the simulation realized with $\varepsilon = 0.5T$ and $\Delta = 0.75T$. Due to the presence of reflected signal, both curves are higher than in the single-path case (see Fig. 3). However, results are quite similar, and the simulated standard deviation is very close to the CR bound limit.

H. P-Path Case

It seems to be possible to extend our approach to the case of more than two paths. The resulting new algorithm we are going

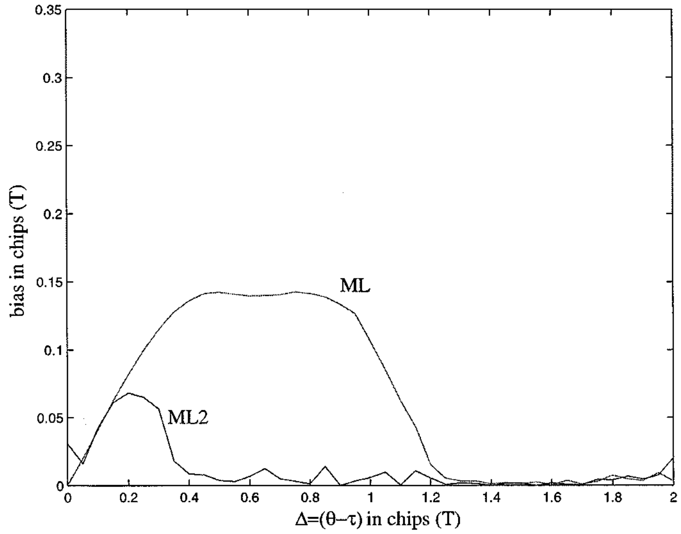

Fig. 8. Bias versus Δ, $\varepsilon = 0.5T$.

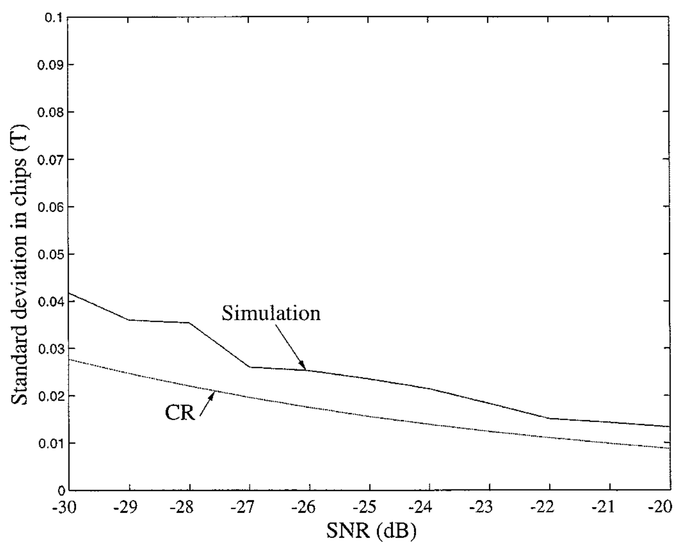

Fig. 9. Standard deviation, $\varepsilon = 0.5T$, $\Delta = 0.75T$.

to describe in this section will be denoted as MLP and is directly deduced from the ML2 method.

The signal model is supposed to be composed by a direct signal component (amplitude A and delay τ), q specular reflected signals (amplitudes a_j and delays τ_j for $j \in [1, p]$), and an additive Gaussian noise

$$s(t) = A(t)c(t-\tau) + \sum_{j=1}^{p} a_j(t)c(t-\theta_j) + b(t). \quad (28)$$

The same hypothesis (see Section V-A) on temporal variations of parameters $A(t)$ and $a_i(t)$ are realized. Both are constant functions on T_m time intervals. In a first approximation, parameter p is supposed to be known.

Writing the likelihood function as we have done in (17) and (18) and the different estimators of time and amplitude parameters in (19) give the following equations in the P-path propagation case:

$$\hat{\tau}_{n+1} = \hat{\tau}_n - \frac{\sum_{i=1}^{N_p}\left(E_i^{(0)}(n) - L_i^{(0)}(n)\right)P_i^{(0)}(n)}{4\sum_{i=1}^{N_p} P_i^{(0)2}(n)}T \quad (29)$$

$$\hat{\theta}_{j,n+1} = \hat{\theta}_{j,n} - \frac{\sum_{i=1}^{N_p}\left(E_i^{(j)}(n) - L_i^{(j)}(n)\right)P_i^{(j)}(n)}{4\sum_{i=1}^{N_p} P_i^{(j)2}(n)}T \quad (30)$$

where $E_i^{(j)}(n)$, $L_i^{(j)}(n)$, and $P_i^{(j)}(n)$ are, respectively, early, late, and prompt channels of $S_i(t-\hat{\theta}_{j,n})$ for $j \in [1, q]$. $E_i^{(0)}(n)$, $L_i^{(0)}(n)$, and $P_i^{(0)}(n)$ are relatives to function $S_i(t-\hat{\tau}_n)$. Therefore, according to (23)

$$S_i^{(0)}(\hat{\tau}) = \int_{(i-1)T_m}^{iT_m} \left[s(t) - \sum_{l=1}^{p} \hat{a}_l c\left(t - \hat{\theta}_l\right)\right] c(t-\hat{\tau})dt \quad (31)$$

and for $j \in [1, p]$

$$S_i^{(j)}\left(\hat{\theta}_j\right) = \int_{(i-1)T_m}^{iT_m} \left[s(t) - A_i c(t-\hat{\tau}) - \sum_{l=1, l\neq j}^{p} \hat{a}_l c\left(t - \hat{\theta}_l\right)\right] c(t-\hat{\theta}_j)dt. \quad (32)$$

These last equations prove that amplitudes need to be estimated to guarantee estimation of delays. As for the two-path propagation case, there is only one parameter (direct signal delay τ) useful to the positioning, but all other amplitudes and delays parameters of model (28) must be also estimated.

Amplitudes $A_{i,n+1}$ and $a_{i,n+1}$ can be estimated by solving the following system:

$$MX_i = Y_i \quad (33)$$

where $X_i = [\hat{A}_i, \hat{a}_{i,j}]_{1\le j\le P}$ and $Y = [S_i(\tau), S_i(\theta_j)]_{1\le j\le P}$ are vectors of dimension $(P+1)N_P \times 1$. The symmetric $(P+1)x(P+1)$ matrix M is composed by $M(1,j) = \Phi(\hat{\theta}_j - \hat{\tau})$ and $M(i \neq 1, j \neq 1) = \Phi(\hat{\theta}_i - \hat{\theta}_j)$.

The MLP algorithm has the same functioning principle as the ML2 algorithm proposed in Section IV. At time $n+1$, the N_P amplitudes $A_{i,n+1}$ are estimated, thanks to (33), which uses the precedent delays estimations $\hat{\tau}_n$ and $\hat{\theta}_{i,n}$. Then, the delay estimators $\hat{\tau}_{n+1}$ and $\hat{\theta}_{i,n+1}$ are calculated thanks to (29) and (30). Conditions on reflected signal delays θ_j are the same as in the two-path propagation case, and all delay estimators are forced to be in the time interval $[\tau, \tau + 1.5T]$ since at this sector, the positioning precision is not affected by these reflected signals.

Moreover, further conditions on the estimators $\hat{\theta}_i$ are fixed, and they guarantee the invertibility of the matrix M.

$$\hat{\tau} + (i-1)\frac{1.5T}{P} \le \hat{\theta}_i \le \hat{\tau} + i\frac{1.5T}{P}. \quad (34)$$

Indeed, the number P of reflected signals is unknown and difficult to estimate. We suggest setting to arbitrary the value of P in the procedure by overestimating the actual value. Note that it is not useful to greatly increase the value of P since two reflected signals very close in time can be estimated as only one without perturbation on the direct path delay estimation [10]. In [10], we have shown that a two-path model is often sufficient. As a result, P should not be taken higher than 5 or 6. Moreover, in order to prevent spurious path estimation, we set a threshold on the direct path to estimated path to take it into account. From simulations, the threshold was set to 12 dB. Below the threshold, the amplitudes a_i are put to 0 so that the corresponding path is not taken into account.

VI. Conclusion

In this paper, we have addressed the problem of the propagation delay estimation for a GPS signal in the presence of multipath propagation. Since the usual early–late estimator derived by the ML principle in the single-path case is biased when there is multipath propagation, we have derived the ML iterative etimator for the case of two-path propagation (ML2) by reusing the current architecture. The ML2 good performances have been illustrated on simulations and by derivation of the corresponding CR bound. Finally, the algorithm has been extended to the general case of P-path propagation (with $P > 1$), where P should represent a small number of propagation beams (each beam modeling a cluster of propagation paths). The new receiver complexity is at most six times that of the early–late receiver.

Appendix

A. Fisher's Information Matrix in Single-Path Propagation

Since the estimation is done using sampled values

$$p(s,[\tau,\underline{A}]) \propto \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{N_p}\sum_{l=1}^{N}(s(lT_e)-A_ic(lT_e-\tau))^2\right\} \quad (35)$$

The Karunen–Loeve decomposition [7] allows a return to the continuous time expression since

$$\sum_{l=1}^{N}(s(lT_e)-A_ic(lT_e-\tau))^2 = \frac{1}{T_e}\int_{(i-1)T_m}^{iT_m}(s(t)-A_ic(t-\tau))^2dt.$$

The Fischer information matrix F is defined by

$$\begin{aligned}F &= -\boldsymbol{E}[\nabla_{(\tau,\underline{A})}[\log p(\underline{s},[\tau,\underline{A}])]]\\ &= -\frac{1}{\sigma^2}\mathbf{E}[\nabla_{(\tau,\underline{A})}[V(\tau,\underline{A})]] \quad (36)\end{aligned}$$

where $\mathbf{E}[.]$ is the mean expectation, and $\nabla_{\mathrm{x}}[.]$ is the second derivative matrix with component i,j equal to $\partial[.]/\partial x_i\partial x_j$. Denoting $b(t)=s(t)-A_ic(t-\tau)$, we get

$$\frac{\partial^2V(.)}{\partial\tau^2} = \sum_{i=1}^{N_p}\frac{A_i}{T_e}\times\int\left(\frac{\partial^2c(t-\tau)}{\partial\tau^2}b(t)-A_i\left(\frac{\partial c(t-\tau)}{\partial\tau}\right)^2\right)dt \quad (37)$$

$$\frac{\partial^2V(.)}{\partial A_i\partial\tau} = \frac{1}{T_e}\int\frac{\partial c(t-\tau)}{\partial\tau}(-b(t)+A_ic(t-\tau))dt \quad (38)$$

$$\frac{\partial^2V(.)}{\partial A_i^2} = -\frac{1}{T_e}\int c^2(t-\tau)dt. \quad (39)$$

Since the random noise distribution is independent from the other parameters, the mean expectation of the terms involving $b(t)$ are null. Moreover, $c(t-\tau)$ can be expressed as the low-pass (bandwidth B) filtered Gold code $G(kT_e-\tau)$, and we will consider the following approximation:

$$\begin{aligned}c(t-\tau) &= \int_{-B/2}^{B/2}\bar{G}(f)e^{2\pi jf(t-\tau)}df\\ &\approx B\int_{-\infty}^{+\infty}G(t-\tau-v)\mathrm{sinc}(\pi Bv)dv \quad (40)\end{aligned}$$

which is true since $BT_m \gg 1$ ($T_m = 1$ ms and $B \approx$ MHz in the case of actual GPS application). Defining $\Phi(\tau)=\int_{[T_m]}G(t)G(t-\tau)dt$, the mean value of (39) becomes

$$\begin{aligned}-\mathbf{E}\left[\frac{\partial^2V(.)}{\partial A_i^2}\right] &\approx \frac{B^2}{T_e}\int\int\Phi(v_1-v_2)\mathrm{sinc}(\pi Bv_1)\\ &\quad\times\mathrm{sinc}(\pi Bv_2)dv_1dv_2 \quad (41)\\ &= \frac{1}{T_e}\int_{-B/2}^{B/2}\bar{\Phi}(f)df \quad (42)\end{aligned}$$

where $\bar{\Phi}(f)$ is the Fourier transform of autocorrelation function $\Phi(t)$. Accordingly, (37) and (38) can be expressed as

$$-\mathbf{E}\left[\frac{\partial^2V(.)}{\partial\tau^2}\right] \approx \frac{4\pi^2}{T_e}\left(\sum_{i=1}^{N_p}A_i^2\right)\int_{-B/2}^{B/2}f^2\bar{\Phi}(f)df \quad (43)$$

$$-\mathbf{E}\left[\frac{\partial^2V(.)}{\partial A_i\partial\tau}\right] \approx \frac{2\pi}{T_e}jA_i\int_{-B/2}^{B/2}f\bar{\Phi}(f)df. \quad (44)$$

Assuming that $\bar{\Phi}(f)$ is even, (44) becomes 0.

With the notations in (15) and $I_{N_p\times N_p}$ the identity matrix of size $(N_p\times N_p)$, the expression of the diagonal Fischer information matrix is the diagonal matrix in (14).

B. Fisher's Information Matrix in Two-Path Propagation

Based on two-path propagation model (16), the log-likelihood of the observation vector $\underline{s}$ is proportional to

$$-\sum_{i=1}^{N_p}\int_{(i-1)T_m}^{iT_m}|s(t)-A_ic(t-\tau)-a_ic(t-\theta)|^2\,dt. \quad (45)$$

The mean expectation of the second derivative of the log-value of (45) defines the Fischer information matrix F_2:

$$\begin{aligned}F_2 &= -\mathbf{E}\left[\nabla_{(\tau,\theta,\underline{A},\underline{a})}[\log p(.)]\right]\\ &= -\frac{1}{\sigma^2}\mathbf{E}\left[\nabla_{(\tau,\theta,\underline{A},\underline{a})}[V(.)]\right]. \quad (46)\end{aligned}$$

The terms related to the direct signal have already been given in (42), (43), and (14) in the previous part of the Appendix. In the same way, expressions of the terms related to the reflected signal are the same with the parameters (τ,A_i) replaced by (θ,a_i):

$$-\frac{1}{\sigma^2}\mathbf{E}\left[\frac{\partial^2V(.)}{\partial a_i^2}\right] = \frac{1}{T_e\sigma^2}\int_{-B/2}^{B/2}\bar{\Phi}(f)df = \lambda \quad (47)$$

$$\begin{aligned}-\frac{1}{T_e\sigma^2}\mathbf{E}\left[\frac{\partial^2V(.)}{\partial\theta^2}\right] &= \frac{4\pi^2}{T_e\sigma^2}\sum_{i=0}^{N_p-1}|a_i|^2\int_{-B/2}^{B/2}f^2\bar{\Phi}(f)df\\ &= w'. \quad (48)\end{aligned}$$

Finally, only nondiagonal terms due to crosscorrelation between direct and reflected signals need to be calculated. The only required hypothesis is still that $B \gg 1/T_m$.

$$\begin{aligned}
-\frac{1}{\sigma^2}\mathbf{E}\left[\frac{\partial^2 V(.)}{\partial\tau\partial\theta}\right] =& \frac{4\pi^2}{T_e\sigma^2}\int_{-B/2}^{B/2} f^2\bar{\Phi}(f)e^{-2\pi jf\Delta}df = w'' \\
-\frac{1}{\sigma^2}\mathbf{E}\left[\frac{\partial^2 V(.)}{\partial A_i\partial\theta}\right] =& -2\pi j\frac{a_i}{T_e\sigma^2}\int_{-B/2}^{B/2} f\bar{\Phi}(f)e^{-2\pi jf\Delta}df \\
=& v_i' \\
-\frac{1}{\sigma^2}\mathbf{E}\left[\frac{\partial^2 V(.)}{\partial\tau\partial a_i}\right] =& -2\pi j\frac{A_i}{T_e\sigma^2}\int_{-B/2}^{B/2} f\bar{\Phi}(f)e^{+2\pi jf\Delta}df \\
=& v_i \\
-\frac{1}{\sigma^2}\mathbf{E}\left[\frac{\partial^2 V(.)}{\partial A_i\partial a_i}\right] =& \frac{1}{T_e\sigma^2}\int_{-B/2}^{B/2} \bar{\Phi}(f)e^{-2\pi jf\Delta}df = \mu \quad (49)
\end{aligned}$$

where $\Delta = (\theta - \tau)$.

As a result, F_2 is given by (50), and CR bounds are diagonal terms of F_2^{-1}. The matrix F_2 is invertible as long as $\theta \neq \tau$. Thus, delay and amplitude bounds are, respectively, the diagonal terms of the matrices C_1 and C_2:

$$F_2 = \left[\begin{array}{c|c} W & \underline{V} \\ \hline \underline{V}^T & M \end{array}\right] = \left[\begin{array}{c|c} C_1 & D_1 \\ \hline D_2 & C_2 \end{array}\right]^{-1} \quad (50)$$

where

$$W = \begin{bmatrix} w & w'' \\ w'' & w' \end{bmatrix} \quad (51)$$

$$\underline{V} = \begin{bmatrix} 0 & \cdots & 0 & v_1 & \cdots & v_{N_p} \\ v_1' & \cdots & v_{N_p}' & 0 & \cdots & 0 \end{bmatrix} \quad (52)$$

$$M = \left[\begin{array}{c|c} \lambda I_{N_p\times N_p} & \mu I_{N_p\times N_p} \\ \hline \mu I_{N_p\times N_p} & \lambda I_{N_p\times N_p} \end{array}\right]. \quad (53)$$

Solving equation $F_2F_2^{-1} = I_{(2N_p+2)\times(2N_p+2)}$ allows expression of C_1 and C_2 through (54). Again, invertibility of matrices M and W is guaranteed by $\Delta = (\theta - \tau) \neq 0$

$$\begin{aligned} C_1 &= \left(W - VM^{-1}V^T\right)^{-1} \\ C_2 &= \left(M - V^TW^{-1}V\right)^{-1}. \end{aligned} \quad (54)$$

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