IRREGULAR CHANNEL CODING FOR OFDM TRANSMISSION

D. Declercq, G. Gelle and V. Mannoni

1ETIS ENSEA/UCP/CNRS-8051
6, avenue du Ponceau 95014 Cergy-Pontoise (France)

2LAM-URCA
UFR-Science, Moulin de la Housse, BP 1039, 51687 Reims (France)
e-mail: valerian.mannoni@univ-reims.fr

ABSTRACT
This paper deals with a new channel coding scheme applied to OFDM transmission. Traditional coding methods use regular codes in the sense that each bit participates in the same way to the channel encoding. We propose to use irregular channel encoding with Gallager codes on an OFDM transmission over frequency selective channels. The motivation is that one should make use of the channel shape knowledge at the transmitter for designing the channel code. In some applications, e.g. ADSL transmission, such knowledge is available. Irregular channel encoding does take advantage of the channel shape, and our simulations show a coding gain of more than 4 dB at a bit error rate of $10^{-3}$ for irregular coding compared to regular coding.

1. INTRODUCTION
Multi-Carrier Modulation (MCM) have been received much attention during these last few years, especially for Digital Audio/Video Broadcasting (DAB/DVB) transmission in Europe with the Orthogonal Frequency Division Multiplexing (OFDM) standart [1], or for Asymmetric Digital Subscriber Line (ADSL) with the Discrete Multitone Transmission (DMT). The principle of these modulations is to transform an input data stream into several orthogonal subcarriers which can be used separately. These subcarriers are made orthogonal by using complex exponential functions and adding a cyclic prefix which ensures the orthogonality property, even for dispersive channels. The success of MCM can be explained by the easyness of their implementation with FFT based algorithms and also by their high spectral efficiency due to the use of overlapped subcarriers.

In this paper, we address the problem of coded OFDM transmissions over frequency selective channels. We propose to use Low Density Parity Check Codes (LDPC) introduced by Gallager in 1963 under two implementation schemes: regular and irregular channel coding. We will show that irregular bit protection is well suited to OFDM transmission since the code design takes advantage of the equalization method specific to multicarrier modulations. Gallager codes have been chosen for their good performance close to the Shannon limit, but also because it is easy to make a Gallager code irregular.

Thus, we wish to demonstrate the interest of irregular channel coding for OFDM systems, compared to regular coding.

The paper proceeds as follows: in section 2 the communication system is described. Section 3 presents the regular and irregular Gallager codes with their decoding algorithm based on belief propagation and motivates the use of irregular codes. Section 4 illustrates the performance of our approach through simulations and we conclude in section 5 with a brief discussion and a presentation of future work.

2. MODEL DESCRIPTION

Fig. 1 shows the structure of the communication system. The input data stream $u_k$ consists in information bits that are encoded by one of the LPDC’s encoder presented in section 3. After channel encoding, the resulting sequence is sent to the OFDM transmitter. So, the sequence $X_k$ at the OFDM modulator input is sent into N subcarriers by serial to parallel conversion and is BPSK-mapped on each subcarrier. Then, the signal is transformed by IFFT processing into a time sequence and, after adding a cyclic prefix to ensure orthogonality of the subcarriers, is sent through the channel with impulse response $h_n$. At the receiver, after removing the cyclic prefix and serial to parallel conversion, the FFT transforms the receive sequence $y_n$ in the frequency domain $Y_k$. Then, a frequency domain equalization (MMSE) can be easily applied to $Y_k$ and after parallel to serial conversion the signal $R_k$ is sent to channel decoder. We finally iteratively decode the LDPC noisy codeword to obtain an estimate $\hat{u}_k$ of the input sequence.

3. IRREGULAR GALLAGER CODES

3.1. Brief Presentation of Gallager codes

Gallager block codes are defined by a parity check matrix which is very sparse. The decoding algorithm we
use is Belief Propagation on a factor graph representation of the code. These block codes have been proposed by Gallager in 1963, together with a stochastic decoding algorithm which is very close to belief propagation. MacKay et al. have recently rediscovered and extended LDPC Gallager codes [2] and have shown that Gallager codes can be easily decoded with iterations of belief propagation on the associated factor graph. A factor graph is a bipartite graph, composed with data nodes that represent information (or redundant) bits and function nodes which verify the parity checks. Iterated belief propagation on a factor graph gives the approximate a posteriori density of each information bit, used to take a decision about their values [3].

A Gallager code is regular if each node participates to the same number of check functions and if each check is connected to the same number of nodes. The Parity check matrix has then constant column weight \( t \) and constant row weight \( t_r \). If moreover \( t \) and \( t_r \) are small, the parity matrix is sparse, reason why Gallager codes are named low density (LPDC codes). An irregular Gallager code is simply a sparse code that is not regular. In Fig. 2, we show the factor graph of a rate \( R = 1/2 \) irregular Gallager code: only the connections to the data nodes are irregular while the check nodes are equally connected. We will denote this type of irregular code \( (t_r; \{\mu_i(i)\}) \), where \( \mu_i \) is the proportion of nodes with connectivity \( i \) in the graph. For example, the code described in figure 2 is called \( (7;\{1/7, 1/4, 1/2\}) \). For mode details about irregular codes, see [4].

**Fig. 2.** Factor Graph of a rate \( R = 1/2 \) irregular Gallager code. We have specified the connexion degree of each node in braces.

The sparseness of the matrix ensures that the decoding algorithm performs almost as Maximum A Posteriori (MAP) decoding. However, because of the presence of cycles in the graph, many iterations of a belief propagation schedule are needed to reach a fixed state. The number of iterations necessary to achieve performance close to MAP decoding depends on the connectivity in the graph. If the graph is sparse, only a few iterations \((10 < K < 20)\) are sufficient. We have considered Gallager codes because they are powerful codes \((K2)\), and because it is somewhat easy to build irregular structures of Gallager codes. In the next section, we motivate the use of irregular codes in OFDM transmission and explain how to choose the irregularity in a clever way.

### 3.2. Why irregular codes?

The goal of irregular protection in Multicarrier transmission is that we want to improve the information rate by using the knowledge of the channel transfer function at the transmitter. Our coding scheme is motivated by the two following remarks:

- for rate \( R = 1/2 \) codes one half of the bandwidth is used to transmit the redundant bits. Moreover, in OFDM signaling over frequency selective channels, some sub-carriers have a very small signal to noise ratio (SNR). The information bits should therefore be sent through the sub-bands with the higher SNRs, in order to achieve the better performance,
- a bit that is connected to a large number of check nodes is well protected against the additive noise, because it gets a lot of information coming from the other bits (information or redundant) during the decoding process.

The criterion we used to build our irregular codes is then to assign more branches in the factor graph to the information bits, which are transmitted through the subcarriers with the higher SNRs. In that way, the information sequence is very likely to be error free because \((i)\) it is transmitted through the less noisy part of the bandwidth and \((ii)\) it gets a great benefit from the coding structure.

In this paper, we consider only rate \( R = 1/2 \) codes and we are currently working on codes of any rate (and specially low rates). We have considered 2 different codes in our study: one weakly irregular Gallager code referred as code(1) and one highly irregular Gallager code referred as code(2):

\[
\text{code}(1) : \left( 9; \left\{ \frac{1}{6} (8), \frac{1}{3} (5), \frac{1}{2} (3) \right\} \right)
\]

\[
\text{code}(2) : \left( 12; \left\{ \frac{1}{4} (11), \frac{1}{4} (9), \frac{1}{2} (2) \right\} \right)
\]

The data nodes that have the higher connexion degrees have been placed on the sub-carriers where the channel is not selective. The coding scheme for code(2) and channel Proakis B [5] is given in Fig. 3.
B. The number of subcarrier is taken equal to 1180 regular coding scheme are compared with those corresponding to regular coding scheme. In order to illustrate the validity of our approach, we have analysed the performance of the communication systems in terms of Bit Error Rate (BER) versus signal to noise ratio $E_b/N_0$:

$$\left( \frac{E_b}{N_0} \right)_{dB} = 10 \log_{10} \left( \frac{2(N + L)}{N\sigma^2_b} \right)$$

with $N$ being the codeword length, $L$ the cyclic prefix length and $\sigma^2_b$ the AWGN variance.

All the simulations were done with time invariant dispersive channels. Fig.5 and Fig.6 illustrate the results respectively for a standard ADSL channel with impulse response coefficients : $h=[0.06, 0.72, 0.54, 0.36, 0.18, 0.114, 0.078, 0.054, 0.033, 0.018, 0.012, 0.008, 0.004, 0.002, 0.001]^T$. and for the channel Proakis B. The number of subcarrier is taken equal to 1180 and the length of the cyclic prefix is equal to $L = 12$. The equalization is done in the frequency domain with a Minimum Mean Squared Error (MMSE) criterion which allows a good compromise between noise and Inter Symbol Interference (ISI) minimization and which can be written as:

$$(R_k)_{MMSE} = \frac{H_k^* Y_k}{|H_k|^2 + SNR_k}$$

where $SNR_k$ denotes the signal to noise ratio encountered in the $k^{th}$ subcarrier. Thus, the output of the MMSE equalizer $R_k$ is used to derive the a priori probabilities that are used to initialize the decoding algorithm. Those probabilities are related to the additive Gaussian noise by:

$$\text{Prob}(R_k|X_k) \propto \exp \left( -\frac{(R_k - X_k)^2}{2\sigma^2_b} \right)$$

The decoding of Gallager codes has been performed with 10 iterations of Belief Propagation. We noticed that with more iterations, the improvement is only effective beyond an error probability of $10^{-5}$. Finally, we use the a posteriori probabilities given by the decoder to take a decision about the information bits. The posterior probabilities for the all-zero codeword at $E_b/N_0 = 4.5dB$ are given in figure 4 for the regular code(0) and the irregular code(2). We can notice that the irregularity of the code yield posterior weights close to 1 for the information bits. This is clearly an advantage over the regular code which gives posterior weights close to 0.5 at this $E_b/N_0$.

4. SIMULATION RESULTS

We propose in this section to apply the different code structures described in section 3.2 to encode each block of information bits $t_k$. The results obtained with irregular coding scheme are compared with those corresponding to regular coding scheme. In order to illustrate the validity of our approach, we have analysed the performance of the communication systems in terms of Bit Error Rate (BER) versus signal to noise ratio $E_b/N_0$:

Fig.4. A posteriori probabilities that the transmitted bits are well detected at $E_b/N_0 = 4.5dB$ and for channel Proakis-B. We can see that in the case of the irregular code(2), the high connexion degree of the information bits yields better posterior probabilities.

Fig.5 shows the performance of the three coding schemes for the ADSL Channel. For an error probability equal to $10^{-5}$ we can see an improvement of 1dB for the irregular code(1) and more than 4 dB for the irregular code(2). This shows the importance of choosing a good irregularity configuration. We currently investigate a methodology for optimizing the irregularity pattern of the code with respect to the channel shape. Our results indicate that irregular coding takes a great advantage of the channel knowledge at the transmitter, compared to regular coding schemes. The same remarks hold for the Proakis B channel (see Fig. 6).

5. CONCLUSION AND FUTURE WORK

We have shown on this paper that for OFDM transmission with frequency selective channel knowledge at the transmitter, irregular error protection coding is a good strategy. We have shown that the improvement can be
larger than 4dB at $P_e = 10^{-5}$ when comparing regular versus irregular Gallager codes. From now on, we restricted our study to codes at rate $R = 1/2$, but we will investigate other rates, still with Gallager codes. We truly believe that combining irregular coding and a classical power loading algorithm would yield even better results.

6. REFERENCES


