Block-Markov LDPC Scheme for Half- and Full-Duplex Erasure Relay Channel

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***Abstract*—The asymptotic iterative performance of the *block-Markov encoding scheme*, defined over bilayer LDPC codes, is analyzed. This analysis is carried out for both *half-duplex* and *full-duplex* regimes. For the sake of clarity and simplicity, a transmission over the binary erasure relay channel is assumed. To analyze the iterative performance of the coding scheme, the *asymptotic threshold boundary* $\gamma(\epsilon_1, \epsilon_2)$ is used as a performance measure. It is derived for two sparse-graph ensembles: Block-Markov Bilayer-Expurgated and -Lengthened LDPC ensembles.**

I. Introduction

Cooperative strategies for relay networks have gained considerable attention during recent years, mainly, because they improve the throughput and/or the reliability function. The simplest scenario of cooperation consists of a single-relay which helps communication between the encoder (source) and the receiver (destination). Cover and El Gamal [1] developed the main strategies, known as Decode-and-Forward (DF) and Compress-and-Forward (CF), and showed an upper bound referred to as the cut-set bound. These strategies rely on the block-Markov coding scheme, having a fixed rate at the source. Since this work, a lot of references has appeared but capacity is only known for some special cases (see [2] and references therein).

Although nowadays user cooperation is entering into wireless communication standards, the code design for cooperative scenarios is only being addressed since several years. The design of efficient sparse-graph codes for relay channels started with works by Razaghi and Yu [3], by Valenti and Zhao [4] and by Jacobsen [5]. The authors proposed particular families of sparse-graph codes for the DF transmission protocol. It should be emphasized that the bilayer structure introduced by [3] has been studied intensively since then. This structure gives rise to various ensembles of bilayer LDPC codes [3], [6] and to bilayer ARA codes [7]. It is also important to mention that the proposed coding schemes have been designed for half-duplex regime.

In this paper, we analyze the performance of the *block-Markov DF scheme, based on sparse-graph codes, in both half-duplex and full-duplex regimes*, in the framework of DF transmission protocol. By exploting the block-Markov structure, used in [1], we define convolutional code structure that will be seen by the destination. The structure of one block of the block-Markov coding scheme represents a practical implementation of random coding scheme and can be viewed as a generalized bilayer structure. Assuming sparse-graph codes (in particular, bilayer LDPC codes), we investigate the asymptotic performance of the block-Markov scheme via density evolution. For the sake of simplicity, through this paper we consider only the binary erasure relay channel (BE-RC). However, the presented scheme and its analysis can be generalized to other binary-input symmetric memoryless channels.

The paper is organized as follows. Section II reviews the necessary background on the block-Markov scheme. Section III introduces the bilayer LDPC structure, modified for the block-Markov setting. Section IV presents its asymptotic performance analysis over the binary erasure relay channel (BE-RC). Section VI concludes the paper.

II. Transmission Model

A. Half- and Full-Duplex DF Scheme

Let us first review the DF scheme [1]. In this setting, the source node S broadcasts an encoded message X_1 to the helping node (relay R) and the destination node D. Then R receives a noisy version of X_1, from which R decodes the message and re-encodes it to obtain the relay encoded message X_2. After this X_2 is sent to the destination end. Hence two transmission scenarios are possible: (i) In the half-duplex scenario, D receives source and relay messages separately, (ii) In the full-duplex scenario, D receives a mix of those two messages. It is assumed that D performs joint decoding of X_1 and X_2, although other decoding schedules are possible.

B. Generation of Messages X_1 and X_2

The generation of both messages: X_1 at the source end, and X_2 at the relay end, are based on the block-Markov encoding scheme. Let w_i be the i-th information message. At the source end, w_i is encoded to $X_1(w_i)$ using a code C_S of length n and rate R. The message, sent by the source, is formed as follows

$$X_1(w_i|s_i) = X_1(w_i) \oplus X_2(s_i), \quad (1)$$

where $X_2(s_i)$ is the so called *coset vector*. Indeed, $X_2(s_i)$ itself is a codeword of a code C_R of length n and rate R_0,

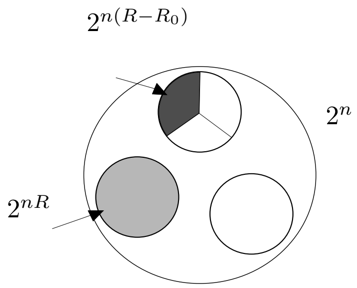

Fig. 1. Code subspace of the Block-Markov scheme.

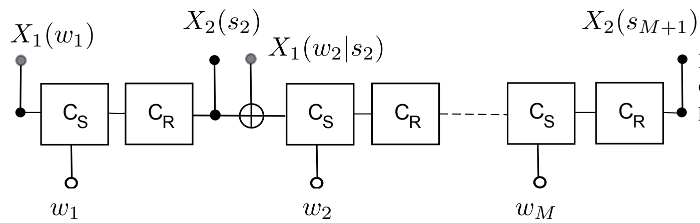

Fig. 2. Block-Markov structure to encode M information messages w_i, $1 \leq i \leq M$.

generated based on the vector s_i. This vector s_i represents the *bin vector*, obtained as follows. Let C_S be partitioned into 2^{nR_0} bins, each containing $2^{n(R-R_0)}$ codewords, and let s denote the bin number. Then s_i denotes the bin number corresponding to $X_1(w_{i-1})$.

The code subspace of the described code structure is given in Fig.1. Large circles are coset codes $C_S \oplus X_2$ of rate R, the dark-grey region represents the bin in a coset code. Note that there are exactly 2^{nR_0} coset codes.

1) Block-Markov Coding Structure: Let the source transmit M messages w_i using the block-Markov relay structure. In this case, the transmission protocol is described as follows:

- Let $s_1 = 0$.
- For i from 1 to M, repeat:
  - $X_1(w_i)$ is generated from C_S at the source end and $X_1(w_i|s_i)$ is broadcasted;
  - R receives $Y_1(i)$, a noisy version of $X_1(w_i|s_i)$, estimates $X_1(w_i)$ and computes s_{i+1}. Then $X_2(s_{i+1})$ is generated from C_R and is sent during the next transmission time;
  - at time i, D receives $Y(i)$, based on $X_1(w_i|s_i)$ and on $X_2(s_i)$.
- For $i = M+1$, R sends $X_2(s_M)$ and S broadcasts $w_{M+1} = 0$.

In the half-duplex regime, the channel output $Y(i)$ is composed of a couple $\{Y_S(i), Y_R(i)\}$, where $Y_S(i)$ and $Y_R(i)$ are respective noisy versions of $X_1(w_i|s_i)$ and of $X_2(s_i)$. In the full-duplex regime, the channel output $Y(i)$ is the output of a multiple-access channel (MAC) with two input messages $X_1(w_i|s_i)$ and $X_2(s_i)$.

The above transmission protocol provides the block-Markov coding structure depicted in Fig.2. C_S blocks represent the coding scheme at the source end and C_R blocks represent the coding scheme at the relay end.

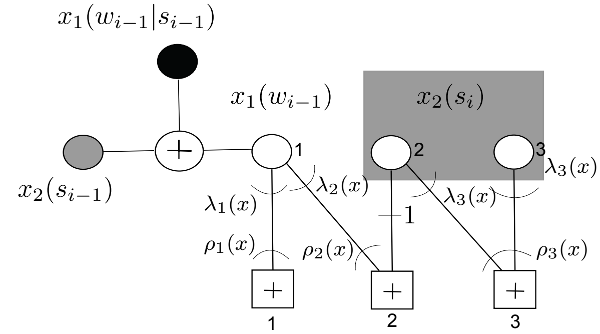

Fig. 3. Structure of a BE-LDPC code, used in the BM scheme. Gray nodes correspond to relay messages, black nodes - to the source message. ⊞s are parity classes and $\oplus$ is the XOR operation.

III. Sparse-Graph Codes

In what follows, C_S and C_R are supposed to be sparse-graph codes. We consider a particular example of a $C_S - C_R$ scheme – bilayer LDPC code, first presented in [3]. Note that in [3] it has been assumed that the destination is able to decode the bin of any codeword coming from the relay, i.e. the relay-destination link is error-free. In this case, to describe the coding scheme, one needs to describe C_S and only the part of C_R, generating the bin of the source's codeword. [3] proposes two ensembles: the bilayer-lengthened LDPC ensemble (both C_S and the part of C_R are LDPC codes), and the bilayer-expurgated LDPC ensemble (C_S is an LDPC code and the part of C_R is an LDGM code).

The aim of our paper is to analyze the behavior of those bilayer LDPC families in the full-duplex regime, as they are naturally first candidates to be considered. Note that, in our case, no assumption on the error-free relay-destination link is made, and one should provide the full description of C_R. Without loss of generality, we still call the code structures bilayer-expurgated (BE) and bilayer-lengthened (BL) LDPC codes. Now let us define them in the block-Markov (BM) setting.

A. Block-Markov Bilayer-Expurgated LDPC Codes (BM-BE-LDPC)

The protograph of a BE-LDPC code is given in Fig. 3. Here, the structure given by $\bigcirc_1$ and $⊞_1$ represents an LDPC code at the source end (C_S LDPC code); the message $x_1(w_{i-1})$ is its codeword. The structure given by nodes $\bigcirc_1 - \bigcirc_2$ and $⊞_2$ is an LDGM code at the relay end, generating the bin message (first part of C_R). Finally, nodes $\bigcirc_2 - \bigcirc_3$ and $⊞_3$ represent an LDPC code, used to generate the message $x_2(s_i)$ (second part of C_R). Each edge class of the protograph is described by left and right degree distributions $\lambda(x)$ and $\rho(x)$.

Note that Fig.3 is a detailed representation of the $\oplus - C_S - C_R$ structure in Fig.2. By putting the BE-LDPC and BM structures together, one obtains a convolutional structure over BE-LDPC codes referred to as a BM-BE-LDPC code.

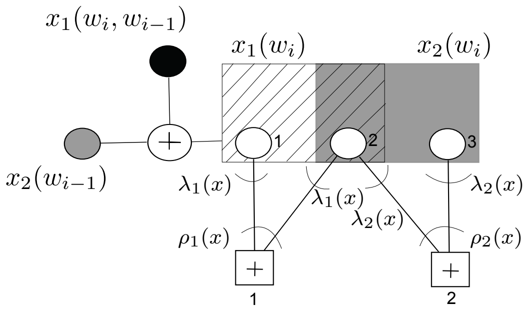

Fig. 4. Structure of a BL-LDPC code, used in the BM scheme. Gray nodes correspond to relay messages, black nodes - to the source message. ⊞s are parity classes and ⊕ is the XOR operation.

B. Block-Markov Bilayer-Lengthened LDPC Codes (BM-BL-LDPC)

For BL-LDPC codes, the coset vector x_2 of the i-th block is generated based on w_i only, which is the information part of the encoded message $x_1(w_i)$.Thus, BL-LDPC codes can be viewed as a particular case of BE-LDPC codes. The C_R code of a BL structure is an LDPC code.

Fig.4 presents the protograph of the BL-LDPC code. The structure given by $\bigcirc_1$, $\bigcirc_2$ and $\boxplus_1$ represents the C_S LDPC code ($\bigcirc_1$ is the class of variable nodes corresponding to w_i). The structure given by $\bigcirc_2$, $\bigcirc_3$ and $\boxplus_2$ is the C_R LDPC code. The transmitted source message is generated as $x_1(w_{i-1}, w_i) = x_2(w_{i-1}) \oplus x_1(w_i)$. In Fig.4, the message $x_1(w_i)$ is represented as the hatched block.

Similar to BM-BE-LDPC codes, one obtains a convolutional BM-BL-LDPC structure by putting together the BE-LDPC structure of Fig.3 and the BM structure of Fig.2.

IV. Analysis of the BM Coding Scheme over the Binary Erasure Relay Channel (BE-RC)

Let us consider the asymptotic performance of the coding scheme from Section II, using bilayer LDPC codes as codes for each $C_S - C_R$ block of the BM encoding structure. Both half- and full-duplex cases are considered.

The analysis is made by assuming the binary erasure relay channel. For simplicity, we assume that the relay always decodes $X_1(w_i)$ successfully, to make density evolution operations over the block-Markov structure easy to define.

A. Binary Erasure Relay Channel

In the half-duplex case, one simply assumes $Y_S(i)$ to be the output of a binary erasure channel (BEC) with probability ϵ_1 and $Y_R(i)$ to be the output of a BEC with probability ϵ_2. For the full-duplex case, the situation is more involved and a MAC model should be defined. As a particular example, we assume the MAC channel it to be sum modulo-2 multiple access channel, as presented in [8]. Clearly, other models of BE-MAC channels are also possible, but the chosen channel model is the closest approximation to the Gaussian MAC channel. Our channel model is given in Fig.5. Here channel input messages are binary and $\oplus$ is the XOR operation.

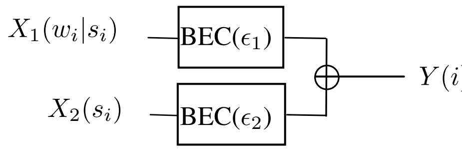

Fig. 5. Model of the binary erasure MAC channel.

As $X_1(w_i|s_i) = X_1(w_i) \oplus X_2(s_i)$ for the BM scheme,

$$Y(i) = \begin{cases} X_1(w_i), & \text{w.p. } (1-\epsilon_1)(1-\epsilon_2); \\ X_1(w_i|s_i), & \text{w.p. } (1-\epsilon_1)\epsilon_2; \\ X_2(s_i), & \text{w.p. } \epsilon_1(1-\epsilon_2); \\ ?, & \text{w.p. } \epsilon_1\epsilon_2. \end{cases} \quad (2)$$

Proposition 1: An achievable rate of the full-duplex BE-RC is given by

$$R \leq \min\{1-\epsilon_1\epsilon_2, 1-\epsilon\epsilon_1\}. \quad (3)$$

Proposition 2: An outer bound on capacity of the half-duplex BE-RC is given by

$$R \leq \begin{cases} \alpha(1-\epsilon_1) + \bar{\alpha}(1-\epsilon_2) & \text{if } \alpha(1-\epsilon) \geq R, \\ \alpha(1-\epsilon_1) & \text{otherwise,} \end{cases} \quad (4)$$

where ϵ is the erasure probability of the source-relay $(S-R)$ link and $0 \leq \alpha \leq 1$ is a coupling parameter that describes the time division between the source and the relay.

B. Analysis of the BM-BE-LDPC Ensemble

Consider the code ensemble defined in Section III-A. Let ϵ_S and ϵ_R be respective bit erasure probabilities, associated with $x_1(w_{i-1})$ and $x_2(s_i)$. ϵ_S and ϵ_R are both explicit functions of (ϵ_1, ϵ_2), depending on the transmission scenario.

Let us define two supplementary functions:

$$\begin{aligned} \rho_j^{(+)}(x,y) &= \sum_i \rho_i \left(\frac{(1-R_j)i}{i-1}\right)^{i-1} \sum_{j=0}^{i-1} \begin{pmatrix} i-1 \\ j \end{pmatrix} \\ &\quad \cdot \left(\frac{R_j i - 1}{(1-R_j)i}\right)^j x^{\circledast j} y^{\circledast (i-j-1)}, \\ \rho_j^{(-)}(x,y) &= \sum_i \rho_i \left(1 - \frac{R_j}{i-1}\right)^{i-1} \sum_{j=0}^{i-1} \begin{pmatrix} i-1 \\ j \end{pmatrix} \\ &\quad \cdot \left(\frac{R_j i}{(1-R_j)i - 1}\right)^j x^{\circledast j} y^{\circledast (i-j-1)}, \end{aligned}$$

where R_j is the fraction of edges corresponding to y. Then density evolution equations for the BE-LDPC ensemble are[1]:

$$\begin{aligned} u_0 &= \epsilon_S \lambda_1(1-\rho_1(\bar{u}_0))\tilde{\lambda}_2(1-\bar{u}_2\rho_2(\bar{u}_1)); \\ u_1 &= \epsilon_S \tilde{\lambda}_1(1-\rho_1(\bar{u}_0))\lambda_2(1-\bar{u}_2\rho_2(\bar{u}_1)); \\ u_2 &= \epsilon_R \tilde{\lambda}_3(1-\rho_3^{(+)}(\bar{u}_3,\bar{u}_4)); \\ u_3 &= \epsilon_R(1-\bar{u}_1\rho_2(\bar{u}_1))\lambda_3(1-\rho_3^{(+)}(\bar{u}_3,\bar{u}_4)); \\ u_4 &= \epsilon_R \lambda_3(1-\rho_3^{(-)}(\bar{u}_3,\bar{u}_4)), \end{aligned}$$

where $\tilde{\lambda}_i(x) = \lambda_i(x)x$, $i = 1, 2, 3$. For some fixed ϵ_S and ϵ_R, let $\hat{u} = (\hat{u}_0, \dots, \hat{u}_4)$ be the largest solution of this system of

[1] Here $\bar{u} = 1 - u$.

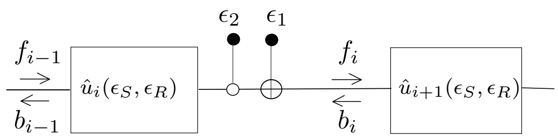

Fig. 6. Forward and backward scheduling for the half-duplex BM scheme.

equations, such that $0 < u_i \leq 1$ for all $i = 0, \dots, 4$. $\hat{u}(\epsilon_S, \epsilon_R)$ is called the *fixed point of the density evolution, corresponding to ϵ_S and ϵ_R.*

1) Half-Duplex: Let us define the density evolution for the half-duplex BM-BE-LDPC scheme. In this case, messages $x_1(w_i|s_i)$ and $x_2(s_i)$ are received separately with erasure probabilities ϵ_1 and ϵ_2 respectively(see Fig.6). We define the following scheduling of iterative decoding of the block-Markov structure with M blocks: forward and backward updates are performed over the block-Markov structure, and the final decision is made for messages $x_1(w_i)$ for all $1 \leq i \leq M$ after some maximal number K of the updates. Note that the forward/backward update for some block i is based on the fixed point estimation $\hat{u}_i$, obtained after an iterative decoding inside the block. Let us define forward and backward updates:

• Initialize $f_i^0 = 1$ for $1 \leq i \leq M - 1$ and $f_i^0 = 0$. Similarly, initialize $b_i^0 = 1$ for $1 \leq i \leq M - 1$ and $b_i^M = 0$.

• For $k > 0$ and a block i, $1 \leq i \leq M$, compute

$$f_i^k = 1 - (1 - \epsilon_1)(1 - \epsilon_2 z), \quad (5)$$
$$b_{i-1}^k = \tilde{\lambda}_1(1 - \rho_1(1 - \hat{u}_0))\tilde{\lambda}_2(1 - (1 - \hat{u}_2)\rho_2(1 - \hat{u}_1)), \quad (6)$$

with

$$z = R_3\tilde{\lambda}_3(1-\rho_3^{(+)}(1-\hat{u}_3, 1-\hat{u}_4))[1-(1-\hat{u}_1)\rho_2(1-\hat{u}_1)] + (1 - R_3)\tilde{\lambda}_4(1 - \rho_3^{(-)}(1 - \hat{u}_3, 1 - \hat{u}_4)), \quad (7)$$

where the fixed point $\hat{u} = (\hat{u}_0, \dots, \hat{u}_5)$ at the forward stage is computed for

$$\epsilon_S = f_{i-1}^k \quad \text{and} \quad \epsilon_R = \epsilon_2(1 - (1 - \epsilon_1)(1 - b_i^{k-1})), \quad (8)$$

and the fixed point $\hat{u}$ for the backward stage is computed for

$$\epsilon_S = f_{i-1}^{k-1} \quad \text{and} \quad \epsilon_R = \epsilon_2(1 - (1 - \epsilon_1)(1 - b_i^k)). \quad (9)$$

Let u^* be the fixed point of the BM structure, namely $u^* = \frac{1}{M}\sum_{i=1}^M \hat{u}_i$, where $\hat{u}$'s are evaluated for $k \to \infty$.

We define the iterative convergence region and the threshold boundary.

Definition 1: The iterative convergence region $\Gamma(\epsilon_1, \epsilon_2)$ is the set of all points (ϵ_1, ϵ_2) such that $\min(u_0^*, u_1^*) = 0$.

Definition 2: The threshold boundary $\gamma(\epsilon_1, \epsilon_2)$ is the set of points that are located on the boundary of $\Gamma(\epsilon_1, \epsilon_2)$.

To estimate the error performance of a BM ensemble, one should compare its threshold boundary with the cpacity bound developed in Section IV-A.

2) Full-Duplex: In this case, a mix of messages $x_1(w_i|s_i)$ and $x_2(s_i)$ is received. By following the model of the BE-MAC channel of Section IV-A, forward and backward updates for the iterative decoding are described by:

For $k > 0$ and the block i $(1 \leq i \leq M)$, compute

$$f_i^k = \epsilon_1 + (1 - \epsilon_1)\epsilon_2 z, \quad (10)$$
$$b_{i-1}^k = \tilde{\lambda}_1(1 - \rho_1(1 - \hat{u}_0))\tilde{\lambda}_2(1 - (1 - \hat{u}_2)\rho_2(1 - \hat{u}_1)) \quad (11)$$

with z given in (7), where the fixed point $\hat{u}$ at the forward stage is computed for

$$\epsilon_S = f_{i-1}^k \quad \text{and} \quad \epsilon_R = \epsilon_1\epsilon_2 + \bar{\epsilon}_1\bar{\epsilon}_2 + \bar{\epsilon}_1\epsilon_2 b_i^{k-1}, \quad (12)$$

and the fixed point $\hat{u}$ for the backward stage is computed for

$$\epsilon_S = f_{i-1}^{k-1} \quad \text{and} \quad \epsilon_R = \epsilon_1\epsilon_2 + \bar{\epsilon}_1\bar{\epsilon}_2 + \bar{\epsilon}_1\epsilon_2 b_i^k. \quad (13)$$

The threshold boundary $\gamma(\epsilon_1, \epsilon_2)$ for the full-duplex regime is defined exactly as in the previous case.

C. Density Evolutuion for the BM-BL-LDPC Ensemble

Similarly to the case of BM-BE-LDPC codes, let us define density evolution operations for the BM-BL-LDPC ensemble and to determine $\Gamma(\epsilon_1, \epsilon_2)$.

First consider some block i. Density evolution equations for a BL-LDPC code as a function of ϵ_S and ϵ_R are given by:

$$u_0 = \epsilon_S\lambda_1(1 - \rho_1^{(-)}(\bar{u}_1, \bar{u}_0));$$
$$u_1 = \epsilon_S\epsilon_R\lambda_1(1 - \rho_1^{(+)}(\bar{u}_1, \bar{u}_0))\tilde{\lambda}_2(1 - \rho_2^{(+)}(\bar{u}_2, \bar{u}_3));$$
$$u_2 = \epsilon_S\epsilon_R\tilde{\lambda}_1(1 - \rho_1^{(+)}(\bar{u}_1, \bar{u}_0))\lambda_2(1 - \rho_2^{(+)}(\bar{u}_2, \bar{u}_3));$$
$$u_3 = \epsilon_R\lambda_2(1 - \rho_2^{(-)}(\bar{u}_2, \bar{u}_3)).$$

As before, $\hat{u} = (\hat{u}_0, \dots, \hat{u}_3)$ denotes the fixed point of those equations for some given ϵ_S and ϵ_R.

1) Half-Duplex: Forward and backward updates in the half-duplex case are given by

$$f_i^k = 1 - (1 - \epsilon_1)(1 - \epsilon_2[R_2 z + (1 - R_2)\tilde{\lambda}_2(1 - \rho_2^{(+)}(1 - \hat{u}_2, 1 - \hat{u}_3))] \quad (14)$$
$$b_{i-1}^k = R_1 z + (1 - R_1)\tilde{\lambda}_1(1 - \rho_1^{(+)}(1 - \hat{u}_1, 1 - \hat{u}_0)) \quad (15)$$

with

$$z = \tilde{\lambda}_1(1 - \rho_1^{(+)}(1 - \hat{u}_1, 1 - \hat{u}_0))\tilde{\lambda}_2(1 - \rho_2^{(-)}(1 - \hat{u}_2, 1 - \hat{u}_3)) \quad (16)$$

and ϵ_S and ϵ_R are given by (8) and (9) for the forward and backward case, respectively.

2) Full-Duplex: The backward update is simply given by (15). The forward update is

$$f_i^k = \epsilon_1 + (1 - \epsilon_1)\epsilon_2[Rz + (1 - R)\tilde{\lambda}_1(1 - \rho_1^{(+)}(1 - \hat{u}_1, 1 - \hat{u}_0)]). \quad (17)$$

In (17), z is given by (16) and ϵ_S and ϵ_R are given by (12) and (13) for the forward and the backward case, respectively.

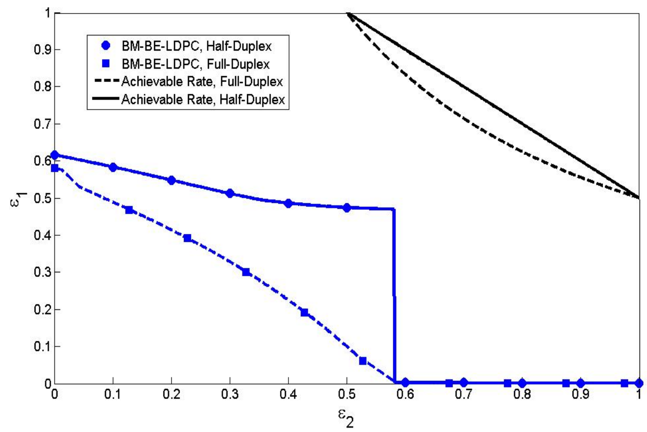

Fig. 7. Threshold boundaries $\Gamma(\epsilon_1, \epsilon_2)$ of BM-BE-LDPC ensemble in half- and full-duplex regimes, compared with capacity bounds.

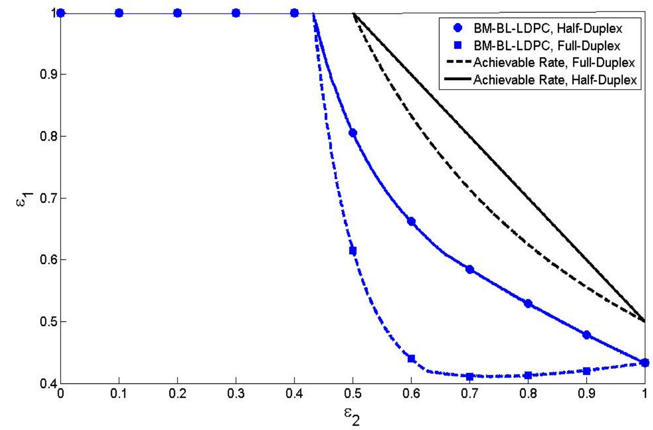

Fig. 8. Threshold boundaries $\Gamma(\epsilon_1, \epsilon_2)$ of BM-BL-LDPC ensemble in half- and full-duplex regimes, compared with capacity bounds.

D. Numerical Example

One would like now to compare threshold boundaries $\Gamma(\epsilon_1, \epsilon_2)$ for BM-BE-LDPC and BM-BL-LDPC codes with capacity bounds in both half- and full-duplex regimes (HD and FD). As the threshold boundaries are difficult to evaluate analytically, numerical evaluations are performed using density evolution equations, as defined in Sections IV-B and IV-C.

For a simple example, let us fix $\lambda_i(x) = x^2$ and $\rho_i(x) = x^5$ for all i ($i = 1, 2, 3$ in the BE case and for $i = 1, 2$ for the BL case), and let $M = 100$. Also, fix the number of forward/backward updates to 100.

Numerical evaluations of threshold boundaries $\Gamma(\epsilon_1, \epsilon_2)$ for block-Markov BE and BL ensembles are given in Fig.7 and Fig.8. Consider $\Gamma(\epsilon_1, \epsilon_2)$ of the BM-BE-LDPC ensemble in Fig.7. One can see that threshold boundaries for bith half- and full-duplex regime are far from capacity bounds. This is due to the presence of the LDGM code with a poor threshold inside each C_R block of the block-Markov structure, and the block-Markov structure accentuates the influence of those LDGM parts. $\Gamma(\epsilon_1, \epsilon_2)$ for the full-duplex case is smoother than $\Gamma(\epsilon_1, \epsilon_2)$ for the half-duplex, which is explained by the difference of two transmission models.

The main difference between the BE and BL LDPC codes is in the fact that $s_i = w_{i-1}$ and thus, there is no LDGM structure in the C_R block. This is beneficial for iterative performance of the scheme – one can see that the threshold boundaries lie closer to outer capacity bounds. Both half- and full-duplex block-Markov threshold boundaries are asymmetric, due to the fact that the source message, corrupted with erasure probability ϵ_1, is in fact a XOR of two codewords. Hence erasures for the source message degrade more the performance than erasures for the relay message. At two extreme points $\epsilon_2 = 1$ and $\epsilon_2 = 1$, which correspond to the absence of relay and to the absence of the source-destination link, the MAC channel acts as an orthogonal channel, and both half-duplex and full-duplex regimes have the same performance.

V. Acknowledgement

M.I. is financed by the DIM COREPHY project.

VI. Discussion

We investigated the asymptotic iterative performance of block-Markov BE-LDPC and BL-LDPC codes over the binary erasure relay channel. Working with such a simple channel model allowed us to derive exactly the iterative convergence region and the threshold boundary of the code families for both half-duplex and full-duplex cases. At our knowledge, this is the first work analyzing coding structures of the full-duplex case. The subject can be further developed with the aim to find optimal code and transmission parameters for this regime.

The calculation of iterative threshold boundary can be extended to other channel models (e.g. the Gaussian channel), even though only the numerical evaluation of this quantity may be possible. Our analysis has been made in the case of the perfect bin estimation at the relay end (good-quality source-relay link). Note that the calculation can be generalized to the case when the estimation error is non-zero. This case is the subject of our future work.

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