Performance Evaluation of Non-Binary LDPC Codes on Wireless Channels

Stephan PFLETSCHINGER\textsuperscript{1}, Alain MOURAD\textsuperscript{2}, Eduardo LÓPEZ\textsuperscript{3}, David DECLERCQ\textsuperscript{4}, Giacomo BACCI\textsuperscript{5}

\textsuperscript{1}Centre Tecnològic de Telecomunicacions de Catalunya (CTTC)
Av. del Canal Olímpic, s/n, 08860 Castelldefels, Spain
Tel: +34 936452915, Email: stephan.pfletschinger@cttc.es

\textsuperscript{2}Samsung Electronics Research Institute
Communications house, South Street, Staines, Middlesex, TW18 4QE, UK
Tel: +44 1784 428600, Email: alain.mourad@samsung.com

\textsuperscript{3}IMEC, Belgium
Tel: +32 16 28115, Email: lopezest@imec.be

\textsuperscript{4}ETIS, ENSEA/Cergy-University/CNRS, F95000, France
Tel: +33 130736627, Email: declercq@ensea.fr

\textsuperscript{5}WISER srl, Via Fiume 23, 57123 Livorno, Italy
Tel: +39 050 2217586, Email: giacomo.bacci@wiser.it

Abstract: This paper evaluates the performance of novel non-binary LDPC codes, which are being developed in the project DAVINCI, and compares them to current solutions employed in IMT-Advanced candidate schemes. After a description of the code properties and the corresponding decoder with reduced complexity, the combination of these $q$-ary codes with QAM is explained. These non-binary LDPC codes are then compared to state-of-art coding schemes in different settings, including MIMO and a realistic channel model. It is found that the DaVinci codes show significant improvements in all cases. Finally, an outline of future work related on integrating these non-binary coding schemes into wireless communication systems is given.

Keywords: Non-binary LDPC codes, IMT-Advanced

1. Introduction

As its name tells, FP7 - DAVINCI project “Design And Versatile Implementation of Non-binary wireless Communications based on Innovative LDPC codes” \cite{1} targets the design of innovative and outperforming non-binary LDPC codes together with their optimization for a versatile implementation on chip. These novel codes are destined for future wireless communication systems aiming at higher spectral efficiency with smaller packet length, such as IMT-Advanced candidates (e.g. IEEE 802.16m and 3GPP LTE-Advanced) and beyond.

The expected performance gain of non-binary LDPC codes comes at the expense of increased hardware complexity, and unsolved technical issues such as non-binary turbo-receivers, or non-binary link adaptation strategies. Consequently, low complexity non-binary LDPC encoders/decoders are being designed together with tailored link level technologies, and will be compared in a realistic framework, using a versatile FPGA implementation and real link using a MIMO-OFDM platform.

With the aim to highlight the outperforming results of DAVINCI codes, performance evaluation and comparison with the advanced reference codes from IMT-Advanced candidates, have been conducted and reported into DAVINCI deliverable D2.2.1 \cite{2}. This paper comes to refine this performance evaluation and summarize the findings, whilst presenting the future work directions towards non-binary link adaptation strategies.
2. Description of Novel Non-Binary LDPC Codes

2.1 Code Design

In the DAVINCI project, we focus on designing a fully non-binary system, including the error correcting codes and decoders. The study and understanding of non-binary LDPC codes are more recent than the binary ones, but sufficient results are available in the literature to help solving an efficient code design for competitive practical applications [3, 4, 5].

In the framework of the DAVINCI project, we propose to use ultra-sparse non-binary LDPC codes designed in a Galois field GF(q) of order q = 64, which corresponds to the largest modulation order considered for wireless communications. The non-binary LDPC codes are described by a Tanner graph with regular and constant connexion degree, with \( d_v = 2 \) edges at the variable node side, and varying parity-check connexion \( d_c \) depending on the desired code rate. To each edge, a non-zero value belonging to the Galois field GF(64) is assigned, in order to define non-binary parity check equations. The choice of the non-zero values is especially important to obtain good performance and requires an optimization strategy. This type of non-binary LDPC code is also referred to as cycle codes and has two main advantages:

- Regular codes with \( d_v = 2 \) are very sparse and the corresponding Tanner graphs have very large girths compared to usual binary codes graphs. As a consequence, iterative decoders show very good performance, especially at small to moderate code lengths. For example, the girth of a binary irregular LDPC code with length \( N = 848 \) bits and rate \( R = 0.5 \) is at most \( g_b = 6 \), while the girth of a NB-LDPC code with same parameters is \( g_{nb} = 14 \) when a good graph construction is used [4, 6].

- As for the code design, it has been shown in the literature [5] that the finite length optimization of non-binary cycle codes can be decomposed into two steps: (i) first build a Tanner graph with the maximum possible girth and the minimum number of cycles with minimal length, then list all ‘short’ cycles and the combination of short cycles which define the smallest trapping sets, (ii) optimize iteratively the choice of the non-zero values on the edges of the cycles, such that the local binary minimum distance computed on the set of cycles and trapping sets is maximized. This optimization procedure allows to gain performance both in the waterfall and the error floor region, compared to a random choice of the Tanner graph structure and of the non-zero values assignment.

The codes proposed for the DAVINCI project have been optimized for finite length using the above mentioned procedure, and are listed with the corresponding girth and global minimum distance properties in Table 1. \( N_{\text{bin}} \) and \( K_{\text{bin}} \) are expressed in bits, \( M \) and \( N \) in symbols.

2.2 Extended Min-Sum Decoder

The performance improvement of non-binary LDPC codes comes at the expense of increased decoding complexity. As in all practical coding schemes, an important feature is the complexity/performance tradeoff, it is very important to try to reduce the decoding complexity of non-binary LDPC codes, especially for high order fields GF(64). The base iterative decoder of non-binary LDPC codes is the Belief Propagation (BP)
Table 1: Non-binary DaVinci Codes in GF(64)

<table>
<thead>
<tr>
<th>Code Rate ( (d_v, d_c) )</th>
<th>( (N_{\text{bin}}, K_{\text{bin}}) )</th>
<th>( (M, N) )</th>
<th>Girth(#mult)</th>
<th>MinDist</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R = \frac{1}{2} ) (2, 4)</td>
<td>(96,48)</td>
<td>(8,16)</td>
<td>8 (36)</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>(288,144)</td>
<td>(24,48)</td>
<td>10 (32)</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>(576,288)</td>
<td>(48,96)</td>
<td>12 (156)</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>(1152,576)</td>
<td>(96,192)</td>
<td>12 (12)</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>(1728,864)</td>
<td>(144,288)</td>
<td>14 (127)</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>(2304,1152)</td>
<td>(192,384)</td>
<td>14 (73)</td>
<td>25</td>
</tr>
<tr>
<td>( R = \frac{2}{3} ) (2, 6)</td>
<td>(96,64)</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(288,192)</td>
<td>(16,48)</td>
<td>8 (200)</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>(576,384)</td>
<td>(32,96)</td>
<td>8 (61)</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>(1152,768)</td>
<td>(64,192)</td>
<td>8 (2)</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>(1728,1152)</td>
<td>(96,288)</td>
<td>10 (262)</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>(2304,1536)</td>
<td>(128,384)</td>
<td>10 (146)</td>
<td>16</td>
</tr>
<tr>
<td>( R = \frac{3}{4} ) (2, 8)</td>
<td>(96,72)</td>
<td>(4,16)</td>
<td>4 (14)</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>(288,213)</td>
<td>(12,48)</td>
<td>6 (64)</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>(576,432)</td>
<td>(24,96)</td>
<td>8 (698)</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>(1152,864)</td>
<td>(48,192)</td>
<td>8 (274)</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>(1728,1296)</td>
<td>(72,288)</td>
<td>8 (136)</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>(2304,1728)</td>
<td>(96,384)</td>
<td>8 (58)</td>
<td>12</td>
</tr>
<tr>
<td>( R = \frac{5}{6} ) (2, 12)</td>
<td>(96,80)</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(288,240)</td>
<td>(8,48)</td>
<td>4 (20)</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>(576,480)</td>
<td>(16,96)</td>
<td>6 (256)</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>(1152,960)</td>
<td>(32,192)</td>
<td>8 (4140)</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>(1728,1440)</td>
<td>(48,288)</td>
<td>8 (3898)</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>(2304,1920)</td>
<td>(64,384)</td>
<td>8 (3760)</td>
<td>9</td>
</tr>
</tbody>
</table>

decoder over the Tanner graph representation of the code. The main difference with the binary BP decoder is that for GF(\( q \)) LDPC codes, the messages from variable nodes to check nodes and from check nodes to variable nodes are defined by \( q \) probability weights, or \( q - 1 \) log-density-ratios. As a result, the decoder complexity scales as \( O(q^2) \) per check node [7], which is too complex for practical applications.

Computing the check node in the Fourier-domain reduces the complexity to \( O(q \log q) \) per check node [4, 8], but adapting the Fourier-domain decoder to practical implementation is tedious due to complicated operators like exponentials or real multiplications.

Recently, sub-optimum decoders based on the generalization of the min-sum decoder have been developed [9, 10]. The core idea of the EMS (extended min-sum) decoder is to only use a limited number of LLR (log-likelihood ratio) values \( n_m \ll q \) both for the storage of messages, and for the computation of symbol and check nodes. This algorithm promises the best complexity/performance tradeoff for LDPC codes in high order fields, and the complexity scales as \( O(n_m, \log(n_m)) \) with \( n_m \ll q \). The performance loss compared to BP decoding is small (around 0.1dB) to negligible, depending on the decoder complexity which is tuned by the value of \( n_m \). Efficient high-rate and parallel implementations of the EMS decoder are under investigation in the DAVINCI project and are presented in the companion paper [11].
2.3 Non-Binary Coding and Modulation

For binary channel coding, the preferred way to combine coding and modulation is BICM (bit-interleaved coded modulation) [12], which can be adapted to non-binary codes by applying a symbol-wise interleaver. The resulting system model is depicted in Figure 1.

The mapping function \( \mu(\cdot) \) assigns symbols out of a \( M\)-QAM constellation \( \mathcal{A}_x \) to the interleaved code symbols \( b_n \) which are taken out of a Galois field of order \( q \). Since the cardinality of both sets is generally not identical, we have to gather \( m_1 \) code symbols and map them onto \( m_2 \) QAM symbols, such that \( q^{m_1} = M^{m_2} \). For common values of \( M \) and \( q = 64 \), this gives the minimum values for \( m_1 \) and \( m_2 \) as denoted in Table 2.

The transmitted QAM symbols are hence given by

\[
x = (x_1 \cdots x_{m_2}) = \mu(b) = (\mu_1(b), \ldots, \mu_{m_2}(b))
\]  

with \( b = (b_0, \ldots, b_{m_1-1}) \). In the following, we only consider the special case of the usual mappings from binary vectors to QAM symbols.

Since the channel decoder applies soft decoding, for each code symbol \( b_n \in \text{GF}(q) \), a LLR vector of \( q \) APP L-values \( L_n = (L_{n,0}, L_{n,1}, \ldots, L_{n,q-1}) \) has to be calculated, which is defined by

\[
L_{n,k} = \ln \frac{P[b_n = \alpha_k | y]}{P[b_n = \alpha_0 | y]}, \quad n = 1, \ldots, m_1, \quad k = 0, 1, \ldots, q - 1
\]  

where \( \alpha_k \) are the GF elements, i.e. \( \text{GF}(q) = \{\alpha_0, \alpha_1, \ldots, \alpha_{q-1}\} \) and \( y = (y_1 \cdots y_{m_2}) \). Under the assumption that all code vectors \( b \) are equiprobable and the channel is memoryless, given by

\[
y_i = h_i x_i + w_i, \quad x_i = \mu_i(b), \quad w_i \sim \mathcal{CN}(0, N_0), \quad i = 1, \ldots, m_2
\]  

we obtain the LLR vector

\[
L_{n,k} = \ln \frac{\sum_{b: b_n = \alpha_k} \exp \left( - \sum_{i=1}^{m_2} \frac{|y_i - h_i \mu_i(b)|^2}{N_0} \right)}{\sum_{b: b_n = \alpha_0} \exp \left( - \sum_{i=1}^{m_2} \frac{|y_i - h_i \mu_i(b)|^2}{N_0} \right)}
\]  

As explained in more detail in [2], this expression simplifies significantly for BPSK, QPSK and 64-QAM, while for 16-QAM no further noteworthy simplification is possible.
3. Performance Comparison with State-of-the-Art Coding Schemes

In this section, the performance of the DaVinci LDPC codes in conjunction with QAM is compared to other state-of-the-art solutions for two simple cases, which facilitate comparison with other solutions, and to an IMT-Advanced candidate system. Appropriate reference channel coding schemes are a duo-binary turbo code (DBTC), which in the WINNER project was found to outperform all other candidate schemes for short to medium codeword lengths [13], and the turbo code defined in 3GPP LTE [14].

3.1 Performance on the AWGN Channel

Figure 2 shows the word error ratio (WER) of the DaVinci, the DBTC and LTE turbo codes codes together with 4, 16, 64-QAM for the codeword lengths $N = 16$ symbols = 96 bit and $N = 384$ symbols = 2304 bit. The code rate is in all cases $R_2 = 1/2$ and the simulated system corresponds to Figure 1 with $h = 1$. As a reference, the Shannon bound on the bit error probability for the BICM channel with the corresponding modulations is also included.

The DaVinci codes perform significantly better for all cases, and the gains are highest for short codewords and higher-order modulation.

3.2 Performance with Spatial Diversity

In Figure 3, both the DaVinci as well as the duo-binary turbo codes are applied on a $2 \times 2$ MIMO system which employs the well-known Alamouti space-time code for spatial diversity. The channel is modelled as uncorrelated Rayleigh fading, i.e. for the coefficients of the channel matrix holds $H_{ij} \sim \mathcal{CN}(0, 1)$.

Again, the relative gain is more pronounced for higher-order modulation, with a gain of nearly 1 dB for 64-QAM at low error rates.

3.3 Performance with Spatial Multiplexing for the LTE Channel

Figure 4 shows the comparison between DaVinci codes and convolutional turbo codes in case of 64-QAM modulation according to the Extended Pedestrian A model (EPA)
propagation conditions specified in [15]. A $2 \times 2$ low correlation antenna configuration has been selected. The performance is expressed in terms of WER. The code rate is in all cases $R = 1/2$. Solid lines correspond to DaVinci codes while dashed lines correspond to the LTE turbo codes. The DaVinci codes show a better performance in all the cases, with higher gains for short codewords.

4. Conclusions and Outlook on Future Work

This paper provided performance evaluation of novel DaVinci non-binary LDPC codes in conjunction with QAM modulations for small to moderate codeword lengths (from 96 to 2304 bits). Performance was first evaluated for two canonical systems (single-antenna AWGN and $2 \times 2$ Alamouti scheme for uncorrelated fading) and compared to the best state-of-the-art scheme (DBTC), and then to LTE turbo convolutional codes on the LTE EPA channel. In all cases, DaVinci codes were shown to perform significantly better, and the gains are found higher for shorter codewords and higher-order modulation. This leads to the conclusion that DaVinci codes are very promising solutions to achieve high spectral efficiency in the challenging but important scenarios of short to moderate codeword lengths and high spectral efficiency.

Future activity is devoted to investigating optimal adaptive combination of modulation and coding to cope with both frequency-flat and frequency-selective channels. This is performed assuming different levels of channel state information (CSI) at the transmitter side, possibly including joint channel and network coding for future mesh networks.

The performance of non-binary-LDPC-based systems can be further increased by adopting novel iterative algorithms for channel estimation and equalization currently under investigation. The ancillary PHY functions, such as carrier/timing synchro-
nization and channel equalization, can be implemented by exploiting the soft output available at the receiver. Using an iterative process that runs in parallel with data decoding [17], it is possible to extend conventional single- and multi-carrier techniques to the case of non-binary DaVinci LDPC codes.

Acknowledgment

This work is supported by INFSCO-ICT-216203 DAVINCI “Design And Versatile Implementation of Non-binary wireless Communications based on Innovative LDPC Codes” (www.ict-davinci-code.eu) funded by the European Commission under the Seventh Framework Programme (FP7).

References


