Regularized Restoration of Scintigraphic Images in Bayesian Frameworks

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Abstract

Scintigraphic imagery is widely used in nuclear medicine and in industrial testing. However, the image quality is very poor due to several degradations: Poisson noise, scattering of gamma photons, and non-stationary impulse response of the gamma detector. The restoration of scintigraphic images is typically an ill-posed inverse problem. In this paper, we propose a restoration method based on the Bayes-Markov approach. The regularization of such a problem is carried out by a Markovian prior. The discontinuity recovery and the restoration of the homogenous areas are improved thanks to the Markov random field (MRF) with an implicit line process. The performance of this approach is shown through the quality measures in terms of contrast around the edges and uniformity in the images, in comparison with two other existing methods.

1. Introduction

Scintigraphic imagery is used in various fields: functional imagery in nuclear medicine, detection of failures or of radioactive sources in non-destructive testing. It clarifies many different phenomena which are barely observable directly, making it a vital tool in gaining otherwise unobtainable information.

We are interested in the problem of restoration of scintigraphic images in nuclear medicine. That is, to reconstruct the projection on a plane of a radio-isotope tracer distribution in an organ, from the gamma rays emitted by the tracer. The grey level variations in different areas give access to functional information on the studied organ.

However, the quality of scintigraphic images is very poor because of several degradation sources: Poisson noise inherent in the photon emission, scattering and attenuation of gamma rays in biological tissues, non-uniformity of the gamma camera spatial impulse response, and bad spatial and energy resolutions of the gamma detector. Together, all of these phenomena cause a non-stationary spatial blur in the observed images, and prevent correct detection of small heterogeneous areas in the organs [1], [2], [3]. Under these conditions, the restoration of scintigraphic images is an ill-posed inverse problem, in the Hadamard’s sense [4], [5].

The resolution of such a problem involves the introduction of a priori constraints (regularization) on the image to be restored.

The aim of our work is to improve the quality of the images and then to increase the detectability of diseased sections in small organs (for example, detection of warm or cold nodules hidden in a thyroid). It is essential not only to make the image as a whole as regular as possible, but also to detect the smallest discontinuities, which may correspond to diseased areas.

Quadratic regularization methods (for example, Gaussian a priori) are not adept at restoring discontinuities.

We propose a new scintigraphic image restoration method based on the Bayes-Markov approach. This dual approach constitutes a unifying and powerful theoretical framework to regularize ill-posed problems: Maximum a Posteriori (MAP) Bayesian estimation enables the combination of different information to obtain the solution, and the main interest of the Markovian model (MM) lies in the consideration of local interactions at the pixel level (local energy). Global energy is defined as a sum of local energies. The model thus defined is global, and intrinsically local. Moreover, Markovian fields enable the introduction of a non-observed process (line process I) to model the discontinuities of the process x being estimated [6].

When a line process is explicit, auxiliary variables (line variables) are introduced to locate and preserve discontinuities. Line variables can be binary or continuous. In the first case and for a first order Markovian model, the line process is defined at each site by a boolean variable which takes the value 1 (or 0) if a discontinuity exists (or not). The a priori energy
concerning \( \mathbf{x} \) and \( \mathbf{l} \) is joint. The image restoration needs to solve a double estimation problem: estimation of an \( \mathbf{x} \) field and of a line process \( \mathbf{l} \). Such an estimation often leads to prohibitive calculation time, which limits its application in clinical routine.

We propose a MM with a Implicit Continuous Line Process (MM-ICLP) inspired by \[7\]. In this case, the potential function of local interactions between pixels is built so as to implicitly preserve discontinuities without introducing auxiliary variables.

The advantages of the MM-ICLP model are numerous:
- The detection of weak transitions as well as the restoration of homogeneous areas are efficiently carried out without introducing additional variables, but only by judiciously choosing the non-convex interaction function associated with the Markov field.
- At the optimization step, an algorithm based on the Graduated Non Convexity (GNC) principle can be applied to an implicit line process. This is no longer possible for an explicit one \[8\], \[9\]. In spite of its sub-optimal solution, the GNC is interesting for several reasons: it is a deterministic relaxation algorithm, fast, independent of the neighbourhood system, not very sensitive to the relaxation scheme, the regularisation parameters, nor the initial point; so its solution is stable.
- We prefer a continuous line process (the hidden variables associated with the line process are continuous values within an interval) rather than a boolean one. The latter can lead to unstable solutions because of the non-continuous relationship between the solution and the data.

We applied the proposed methods and algorithms to scintigraphic images of thyroids. The real data were acquired by a gamma camera from a phantom.

The obtained results are compared to images restored by the Maximum Likelihood method (ML) proposed in \[10\], and by the MAP method with the Gaussian regularization (MAP-Gauss) proposed in \[11\]. The performance of this approach is evaluated using different quality measurements: contrast at discontinuities and uniformity (variance) in the homogeneous areas of images.

The methods and algorithms proposed yielded satisfactory results after a reasonable computation time (reduction of the calculation time by a factor of ten in comparison with the stochastic relaxation algorithm of simulated annealing).

2. Method and algorithm

The image formation process is a linear model with white Poisson noise. Let us note, respectively, \( \mathbf{x} \) and \( \mathbf{y} \) the vectors corresponding to the original and observed images. The number \( Y_i \) of photons detected at each pixel site \( i \) in the observed image follows a Poisson law with parameter \( \mathbb{E}[Y_i] = m_i \), where \( m_i \) represents the average number of photons detected at pixel \( i \) by the gamma camera:

\[
\mathbb{E}[Y_i] = m_i = \sum_{j=0}^{N^2-1} A_{ij} x_j = (\mathbf{Ax})_i
\]  

(1)

Each component \( A_{ij} \) of the \( \mathbf{A} \) matrix can be considered as the probability that a photon emitted from real position \( j \) in the original image is detected at position \( i \) by the gamma camera. \( N \times N \) is the dimension of the image.

Because this matrix is ill-conditioned, the restoration of scintigraphic images is typically an ill-posed inverse problem \[4\]. We propose to solve it in a Bayesian framework. The restored image is the one which maximizes the \textit{a posteriori} probability of the Bayesian rule \[12\]:

\[
p(X=x|Y=y) = p(Y=y|X=x) p(X=x) / p(Y=y)
\]  

(2)

The first term insures the fidelity of the solution to observed data. It only depends on the image formation model:

\[
p(Y=y|X=x) = \prod_{i=0}^{N^2-1} \exp[-(\mathbf{Ax})_i] \]

(3)

The second term reflects the characteristics (\textit{a priori} constraints) of the original object.

In this paper, as \( \mathbf{X} \) is modeled by a Markov random field, the probability density takes the particular form:

\[
p(X=x) = \frac{1}{Z} \exp[-U(\mathbf{x})] \quad \text{with} \quad U(\mathbf{x}) = \alpha \sum_{c \in \mathcal{C}} V_c(\mathbf{x})
\]  

(4)

where \( Z \) is a normalizing constant, \( V_c \) is the local interaction potential function associated with the Markov field, and \( \mathcal{C} \) is a set of cliques associated with the neighbourhood system. In order to model discontinuities and homogeneous areas at once, we built \( V_c \) from the interaction function presented in \[7\], \[8\], \[9\]:

\[
V_c(\mathbf{u}) = \frac{u^2}{u^2 + 3\delta^2}
\]  

(5)

where \( u \) symbolizes the numerical approximations of the horizontal and vertical gradient at each pixel (in the case of a first-order neighbourhood), \( \delta \) is a constant linked with the transition threshold between different areas. \( V_c \) is a particular function. It is concave on \([0, +\infty[\) and has a finite asymptotic behaviour: \( \lim_{u \to \infty} V_c(\mathbf{u}) < \infty \). Such a function implicitly defines a continuous line process which enables the efficient detection of discontinuities without introducing the line variable \[7\].

The maximum of the \textit{a posteriori} probability is obtained by minimizing a weighted sum of two terms:

\[
F(\mathbf{x}) = \sum_{i=0}^{N^2-1} \left[ y_i \ln((\mathbf{Ax})_i) + (\mathbf{Ax})_i \right] + \alpha \sum_{c \in \mathcal{C}} V_c(\mathbf{x})
\]  

(6)

where \( \alpha \) is the regularization parameter.
The proposed method was applied to simulated and real data acquired by a gamma camera.

In the former case, the original image contained two cold nodules 20 mm in diameter located at 6 mm apart, inside a radioactive cylinder. The Width at Half of the Maximum (WHM) of the impulse response is about 23 mm.

The observed image (data) was simulated using the Monte-Carlo method. Figure 1 shows the original and observed images and restorations by different methods: ML, MAP with the Gaussian a priori (MAP-Gauss) and the proposed method with an implicit continuous line process (MAP-ICLP).

The desired solution is the one which minimizes the function $F(x)$:

$$x_{\text{map}} = \text{Argmin}_x F(x).$$

The restored image is the global minimum of this function. Unfortunately, as the hessian is a non-definite positive matrix, the function $F(x)$ is non-convex on $x$. The resolution of the MAP then becomes a variational problem which consists of the minimization of a non-convex function.

Stochastic relaxation methods theoretically converge to the global minimum [13], [14]. Their main drawback is the high computation cost.

A variety of methods based on expectation maximization (EM) have also been proposed. The slow convergence and instability of the EM algorithm is well documented in [15].

We developed a deterministic relaxation algorithm based on the GNC first presented in [8] for image segmentation, then generalized for inverse problems in [9]. The principle of the GNC is to approximate a non-convex function $F(x)$ with a sequence of continuously derivable functions $F_r(x)$ converging to it while $r$ tends to infinity, and with the condition that the initial function $F_0$ is convex.

Unlike the simulated annealing method, the GNC algorithm does not assure convergence to the global minimum of the expression. However, the obtained solution remains close to it. Moreover, this solution is not particularly sensitive to the relaxation scheme nor to the regularization parameter. The computation time is reduced by a factor of ten compared to simulated annealing. It is therefore advisable to apply the GNC algorithm in clinical routine.

Let us note that the GNC can only be used with an implicit line process. This again justifies our choice of the Markovian model with the implicit line process.

3. Results

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The quality of the restored image is evaluated by the amount of contrast at the nodules, and the variance (uniformity) in homogeneous areas. These quality measures are presented in Table 1.

![Figure 1. Simulation of the model containing two nodules and images restored by different methods:](image)

<table>
<thead>
<tr>
<th>Two nodules model</th>
<th>Contrast</th>
<th>Uniformity (variance)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original image</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Observed image</td>
<td>0.02</td>
<td>120.74</td>
</tr>
<tr>
<td>MV</td>
<td>0.18</td>
<td>123.25</td>
</tr>
<tr>
<td>MAP-Gauss</td>
<td>0.41</td>
<td>86.21</td>
</tr>
<tr>
<td>MAP-ICLP</td>
<td>0.71</td>
<td>29.13</td>
</tr>
</tbody>
</table>

One can observe in Table 1 that the image restored by the (MAP-ICLP) gives the closest quality measures (in terms of contrast at the edges and uniformity in homogeneous areas) to those of the original image. This results in improvement to the nodule detectability. The MAP method with the Gaussian regularization gives very smooth images, and the ML method without regularization gives poor quality images.

The last case concerned the restoration of a conical phantom containing two cylindrical nodules 10 mm in
diameter and 5 mm apart. The observed image was acquired by a medical gamma camera. The WHM of the camera impulse response is about 9 mm in this case.

The restored images by three methods (ML, MAP-Gauss and MAP-ICLP) are presented on Figure 2.

![Image with labels](Image)

Figure 2. A conical phantom containing two cylindrical nodules, the real data acquired by a medical gamma camera and images restored by different methods:
- a - Original image
- b - Observed image (real data)
- c - Restoration by ML and EM algorithm
- d - Restoration by (MAP-Gauss)
- e - Restoration by (MAP-ICLP) with GNC.

After carrying out a sequence of tests for different sizes of nodules, and different distances between nodules, we showed in [1] that the images presented in this paper correspond to the limit size and distance of detectability for the impulse response given.

4. Perspectives

The proposed method gives satisfactory results in terms of image quality and computation time. However, we notice that the restoration would be further improved if the energy information concerning the scattering of gamma photons was included in the image enhancement process [16]. Work is currently being conducted to extend the present method to a joint spatial-energy estimation.

References


